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## Acceptance Sampling Plan: A Case Analysis in Automobile Industry

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### ABSTRACT:

The contribution of automobile sector has been conducted as the growing sectors of Indian economy. In 2017, India was recorded as fourth largest market of automobile industry in the world. To test the reliability of any automobile depends on quality standards implementation. The statistical quality control plays an important role to maintain the quality of the automobile product. The acceptance sampling plan is one of the major technique of SQC. We have been introduce SASP for this study.

**Key Words:** Automobile Industry, SQC, ASP, OC, AOQ, ATI.

### 1. INTRODUCTION

The automobile industry is approximately more than four centuries old and in early 17<sup>th</sup> century the first invention was recorded in automobile industry i.e.1769. A great French Engineer, Nicolasan J.Cugnot invented the first three – wheeler, military tractor which is self – powered and developed with the use of steam engine. Thereafter, an automobile powered by a steam engine was developed in U.S. by Oliver Evans and it was succeeded as era of electric carriage. In 1842, Thomas Davenport of the U.S.A and Scotsman Robert Davidson invented more applicable automobiles, making use of non – rechargeable electric batteries Col. Rookes Crompton introduced public transport Wagons strapped to and pulled by imported steam road rollers called steamers. After Second World War in 1945 the automobile industry of other technologically advanced nation such as Japan and certain European nation gained momentum Simpson & Co. established in 1840 were the first to build a steam car and a steam bus in India. American General Motors, a wholly owned Indian subsidiary of started assembly of CKD Trucks and Cars in 1928 in Bombay.



In 1982 MarutiUdyog Limited (MUL) came up as a Government initiative in collaboration with Suzuki of Japan to establish volume production of contemporary models. In the era of globalization, India became a more independent in the field of technologies and the result of first wave of initiatives were consolidated and then in early 1990's another tranche of reforms in automobile sector had been made by Government to invite more companies in this field.

The dream of the journey of automobile industry began to start in 1991 by the announcement of New Industrial Policy delicensing of Automobile Industry by Govt. Of India. This regular growth of the automobile industry in India, the contribution of automobile sector has been counted as the growing sectors of Indian economy. Tata Motors, Maruti Suzuki and Mahindra and Mahindra, are some good automobile manufacturers who have expanded their domestic and international operations. Hyundai, Suzuki, General Motors and other foreign automobile players have set up their base and India is most suitable market as a strong automotive R & D hub. Department of Industrial Policy and Promotion (DIPP), Ministry of Commerce has announced that the period of April 2000 to November 2016 total cumulative FDI inflow in Automobile Industry in India was worth US \$ 7,518 million. Moreover, the passenger vehicles segment grew at 9.71% during April – June 2016, however total commercial vehicles segment recorded an increase of 6.06% year – on – year. According to a report titled , “Strategic Assessment of small and light commercial vehicles in India” by Frost & Sullivan , the Indian small and light commercial vehicle segment is expected to more than double and to grow at 18.5% compound annual growth rate (CAGR) for the next five years. The automobile industry in India has become one of the largest manufacturing sectors all over the world. However, there has been a sudden spurt in the Indian Automobile Industry, especially since last two years. The automobile industry has posted double digits growths in India since then.

The automobile industry started in 1860s and since then it has developed exponentially and now a days it has becoming an integral part of our society. The automobile is a primary requirement for mode of transportation for our society, even the economy of many country depends on automobiles sector. India is fourth largest market of automobile industry in the world in 2017. In year 2017, approximately 92 million motor vehicles were produced worldwide. As a consequence of which automobile

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industry has become one of the world's largest economic sectors by revenue . A regular increase in demand of automobile and globalization has made automobile industry a promising industry towards consumer. Now a days consumers has various trend of options to choose their ranging from economic cars to luxury cars, however certain features such as accessories, handling, capacity, affordability and reliability can shift consumer preferences towards a particular manufacturer.

Automobile manufacturing companies put a lot of effort in improving the reliability and safety of their products in order to attract and retain consumers. Safety for the automobiles themselves, implies that there is no risk of damage. Since safety regulations in the automobile industry is very important. On the other hand, reliability implies the optimum functioning of automobiles as per general standards for a sustained period of time without any malfunction. Generally a major segment of consumers prefer reliable vehicles opposed to the vehicles loaded with all kind of accessories and latest technology.

In order to test the reliability or standard quality item, a number of techniques are available to prevent defective products from reaching the market. Optimising or standardizing manufacturing processes to improve the quality of the product and minimize the proportion of defective items produced is the first criteria. The Statistical Quality Control is also an important part to maintain the quality of the product. It can be divided into two major parts one is Process Control and other is Product Control. The process control is the combination of control charts and six sigma limits. However even when the process is in control, so that the proportion of defective products is low during manufacturing process. In order to test that a lot of finished product do not contain an excessively large proportion of defective products, product control can be met through sampling inspection method. For any manufacture it is not possible to make a 100% inspection due to economic and some other constraints. So, it is feasible to conduct an acceptance sampling plan. The products are divided into defective or non-defective by lot inspection and the quality of the lot defined from the sample fraction defective. The sampling plans employed especially in automobile industry are of *acceptance – rectification* type. The finished products (vehicles) in automobile industry require considerable wealth to produce, therefore it is often easier and cost – effective to rectify the defective products in a sample and return the rectified products back to the sample than to replace them with non – defective ones.

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A sampling plan in which a random sample of size  $n$  is drawn and the number of defectives in the sample are counted, if we count does not exceed  $c$  (a predetermined quantity) then the lot is accepted and all defectives in the sample are either replaced or rectified, else, if the count exceeds  $c$  then the whole lot is inspected and all the defectives in the lot are either replaced or rectified, such a sampling plan is called *Simple Sampling Plan*. This technique came to be used majorly during the Second World War by the armies. The most widely used sampling plans are given by Military Standard (MIL–STD–105E).

## 2. OBJECTIVES OF THE STUDY

The main objectives of this study are:

- (1) To find Operating Characteristic (OC) function of recall data.
- (2) To find Average Outgoing Quality (AOQ) of recall data.
- (3) To find Average Total Inspection (ATI) of recall data.

## 3. METHODOLOGY

Acceptance Sampling (AS) is concerned with inspection and decision making regarding products. AS was one of the major components of the field of statistical quality control, and was used primarily for incoming or receiving inspection. In the other hand, AS is used to make dispositions on accepting or rejecting a lot (or batch) of product that has already been produced. Here, we select a sample from the lot, and some quality characteristics of the units in the sample is inspected.

Consequently, Acceptance Sampling Plan has an important point to remember is that the main purpose of acceptance sampling is to decide whether or not the lot is likely to be acceptable, not to estimate the quality of the lot. According to Schilling an AS plan has much the effect of a lone sniper while its scheme can provide a fusillade in the battle for quality improvement. In a view of ISO standard, an acceptance control chart combines consideration of control implications with elements of acceptance sampling. It is an appropriate tool for helping to make decisions with respect to process acceptance.

Several Acceptance Sampling Plans are available for attributes and variables. The attributes sampling plan is an easy method to identify the defective items and

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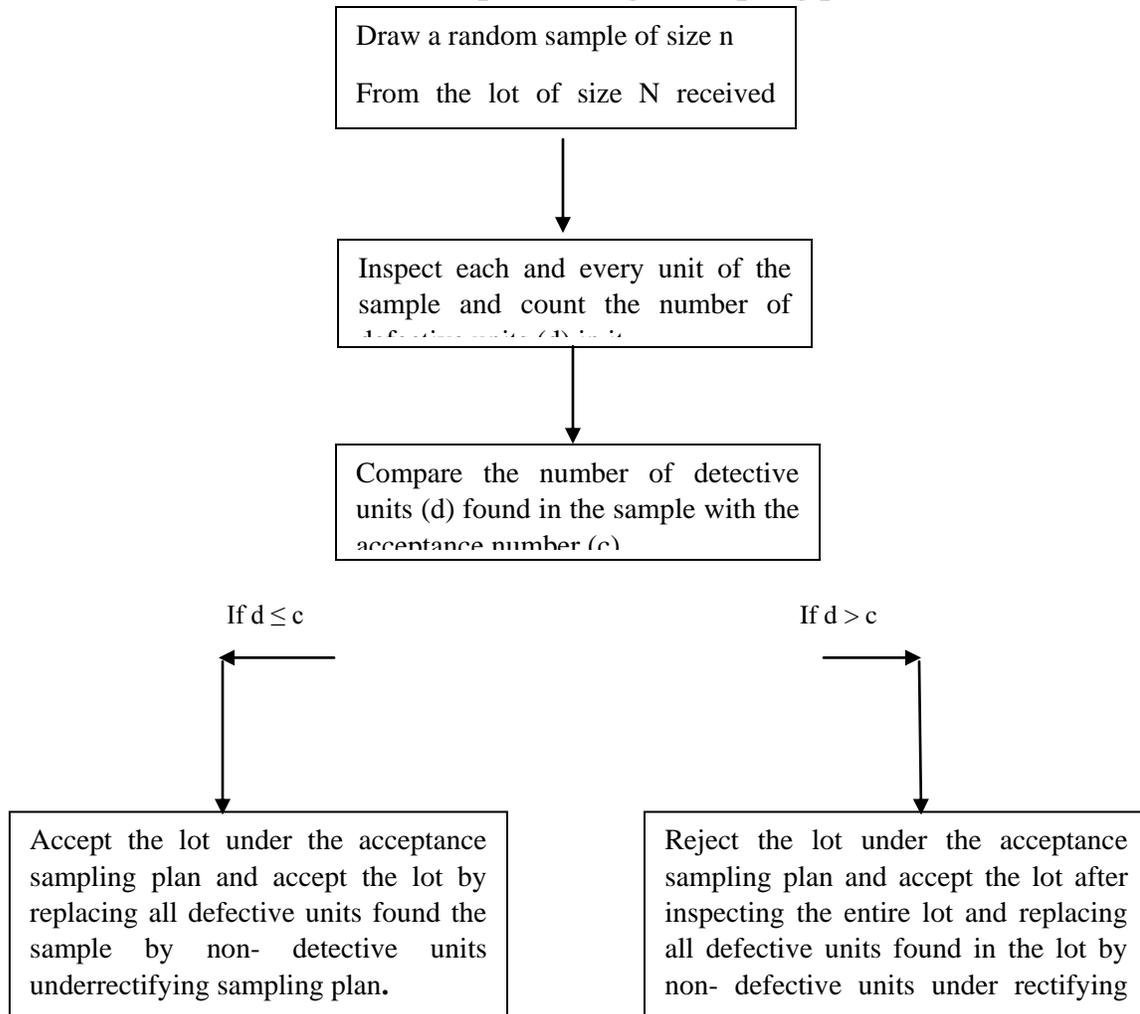


further take a decision about acceptance or rejection of a lot. Variables sampling is designed to predict the value of a given variable and to decide based on measurement values. Thus, statistically valid sampling plan tells us the probability of accepting bad lots and the probability of rejecting goods lots in the manufacturing system. The other classifications are single, double, multiple and sequential sampling plans.

**In this point, some definitions are required which are given as follows:**

- Acceptance Quality Level (AQL) is generally defined as the percent defectives that the plan will accept 95% of the time. Otherwise, lots that are at or better than the AQL will be accepted 95% of the time.
- Rejectable Quality Level (RQL) is the poorest level of quality for consumer.
- Lot Tolerance Percent Defective (LTPD) is generally defined as percent defective that the plan will reject 90% of the time.
- Operating Characteristics (OC) Curve is created by plotting the percent defective versus the matching probabilities of an acceptance. The probability of acceptance is based on the number of samples to be evaluated and the quantity of rejects that are to be allowed.
- Producer's Risk ( $\alpha$ ) is the probability of rejecting the acceptable and it is typically 5% and Consumer's risk ( $\beta$ ) is the probability of accepting a defective sample and it is typically 10%.
- Average Outgoing Quality Level (AOQL) is a simple relationship between quality shipped and quality accepted.
- Average Total Inspection (ATI) is the average number of units inspected per lot, including all units in rejected lots and Average Sample Number (ASN) is the number of samples the receiver has to do.

### Procedure for implementing a sampling plan



### 3.1. OPERATING CHARACTERISTICS (OC) CURVE

The operating characteristics (OC) curve is an important aspect of an acceptance sampling plan. The curve displays the discriminatory power of the sampling plan. That is, it shows are probability that a lot submitted with a certain fraction defective will be either accepted or rejected. For constructing an OC curve, we require the probabilities of accepting a lot corresponding to different quality levels. Therefore, we first compute the

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probability of accepting a lot of incoming quality p for a single sampling plan. In a single sample plan, we accept the lot if the number of defective units (d) in the sample is less than or equal to the acceptance number (c). It means that if X represents the number of defective units in the sample, we accept the lot if  $X \leq c$  i.e.  $X = 0$  or  $1$  or  $2 \dots, c$ .

Therefore, the probability of accepting the lot of income quality p is given by

$$\begin{aligned}
P_x(p) &= [X \leq c] = P [X = 0 \text{ or } 1, \dots \text{ or } c] \\
&= P [X = 0] + P [X = 1] + \dots + P [X = c] \\
&= \sum_{x=0}^c P[X = x]
\end{aligned}$$

From industrial experience that n is usually small for any economically worthwhile production process. Therefore, when sample size n is small compared to lot size (N) i.e. when  $N > n$ . We know that the hyper geometric distribution is approximated by the binomial distribution with parameters n and p where p is the lot quality. It is far easier to calculate the probabilities with the help of the binomial distribution in comparison with the hyper geometric distribution.

Therefore,  $P [X = x] = {}^n C_x p^x (1 - p)^{n-x} \dots \dots \dots (I)$

Thus, the probability of accepting a lot of quality p using binomial approximation

is given by

$$\begin{aligned}
P_x(p) &= \sum_{x=0}^c P[X = x] \\
&= \sum {}^n C_x p^x (1 - p)^{n-x}
\end{aligned}$$

When p is small and n is large such that np is finite, we know that the binomial distribution approaches the Poisson distribution with parameter  $\lambda = np$ . Therefore, the probability of accepting a lot of quality p using the Poisson approximation is given by

$$P_x(p) = \sum_{x=0}^c \frac{e^{-\lambda} \lambda^x}{x!} \dots \dots \dots (II)$$

### 3.2. AVERAGE OUTGOING QUALITY (AOQ)

The average outgoing quality (AOQ) is the average quality level of a series of batches that leave the inspection station, assuming rectifying inspection, after incoming quality lot for inspection at a certain quality level p. The AOQ is not the quality level of a single batch that leaves the inspection station. For instance, a batch with incoming quality level p will leave the inspection station with the same quality level if accepted by



the sample plan. We assume that the sample is a small proportion of the lot, such that if non – conforming items are found in the sample and replaced with confirming once, the quality level of the lot is not significantly affected. Likewise, any other batch with the same incoming quality  $p$  that is rejected by this sampling plan will be screened and so will leave the inspection station with no non – conforming items. (This assumes the screening defects all non – conforming items) The AOQ measures the average quality level of large number of such batches of incoming quality, the proportion non – conforming in the lots, assuming rectification.

Taking  $N$  as the lot size ,  $n$  as the sample size,  $p$  as the incoming lot quality,  $p_a$  as the probability of accepting the lot using the given sample plan, the average outgoing quality is given by  $AOQ = \frac{(N - n) \cdot P \cdot p_a}{N \cdot n}$  .....(III)

### 3.3. AVERAGE SAMPLE NUMBER (ASN)

Two other features that are also useful for any sampling plan are the average sample number (ASN) and the average total inspection (ATI). The average sample number is the expected Single Sampling Plans number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under the acceptance sampling plan. In acceptance single sampling plan, the decision about the acceptance or rejection of a lot is taken on the basis of a single sample that has been inspected. Therefore, the ASN in a single sampling plan is simply the sample size  $n$ . It means that ASN is constant in a single sampling plan. Therefore, in case of acceptance sampling plan  $ASN = n$ , whereas of a corrective sampling plan.

$$ASN = n + [1 - P(p)] \times (N - n) \dots\dots\dots(IV)$$

Where  $P(p)$  = Probability of acceptance

The curve drawn between the ASN and the lot quality ( $p$ ) is known as the ASN curve.

### 3.4. AVERAGE TOTAL INSPECTION (ATI)

The concept of average total inspection (ATI) is considered under rectifying sampling plan in which rejected lots undergo 100% inspection. It is defined as follows: The average number of units inspected per lot under the rectifying sampling plan is called the average total inspection (ATI) .So in a rectifying single sampling plan , the number of units to be inspected will depend on two situations given below:

- (i) If the lot of size  $N$  is accepted on the basis of a sample of size  $n$ , the number of units

inspected is  $n$  and the probability of the accepting the lot is  $P_a$ .

(ii) If the lot is rejected on the basis of a sample, we inspect the entire lot of size  $N$  and the

probability of rejecting the lot is  $(1 - P_a)$ .

Therefore, we can compute the ATI for a single sampling plan as follows:

$$\begin{aligned} \text{ATI} &= \text{Average number of units inspected per lot} \\ &= \sum (\text{inspected number of units} \times \text{probability of taking decision}) \end{aligned}$$

$$\text{ATI} = n \times P_a + N \times (1 - P_a)$$

This can also be written as,

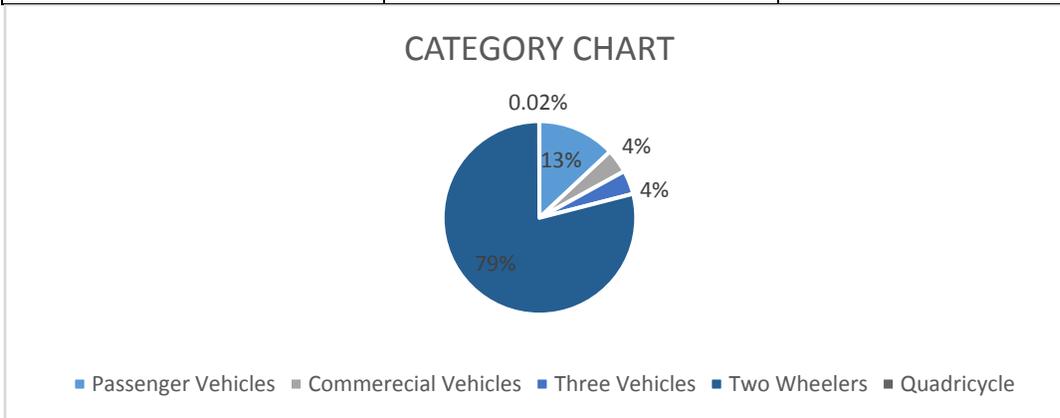
$$\text{ATI} = n + (1 - P_a) (N - n) \dots\dots\dots(V)$$

This curve drawn between ATI and lot quality ( $p$ ) is known as the ATI curve.

#### 4. DATA ANALYSIS

The latest production trends in automobile industry of India are provided below:

CATEGORY	2016 – 17	SHARE
Passenger vehicles	40,28,471	13%
Commercial vehicles	11,12,405	04%
Three vehicles	12,68,833	04%
Two wheelers	2,44,99,777	79%
Quadricycle	5,388	0.02%
<b>Total</b>	<b>3,09,14,874</b>	<b>100%</b>





As per the Society of India Automobile Manufacturers, vehicle recall data for Hyundai Motor India Ltd due to various quality defects for the year 2017 as shown below:

MANUFACTURER'S NAME	TOTAL NO. OF RECALL VEHICLES
Hyundai Motor India Ltd.	16,409

### PERFORMANCE OF SINGLE SAMPLING PLAN IN HYUNDAI MOTOR INDIA LTD.

Based on the operating Characteristics (OC) curve, probability distribution of the existing sampling method is computed and presented graphically. Hyper – geometric distribution, Binomial distribution and Poisson distribution are probability distributions that can be commonly used to find the probability of acceptance. Since Poisson distribution is the most popular, in this study, it is used to compute acceptance sampling plan in Hyundai motors. The shape of an OC – curve is determined primarily by sample size (n) and the acceptance number (c) although there is a small effect of lot size (N). Probability of acceptance can be computed by the following Poisson's Distribution equation.

$$P(X) = \sum (e^{-np} (np)^c / c!)$$

Where n = sample size, c = acceptance number, p = percentage of defectives.

From the table of Automobile Production Trends, it is observed that the total production in India for the year 2016–17 was 3,09,14,874 vehicles. Since the market share of Hyundai Motors India Ltd. is 16% of the total production.

Total production of Hyundai Ltd. =  $0.16 * 3,09,14,874 = 49,46,380 \sim 50,00,000$

i.e. **N = 50,00,000**

Total number of defects (recalls) for Hyundai Ltd in the year 2017= 16,409 – 16,500

This implies,  $P(\text{default}) = 16,500 / 50,00,000$

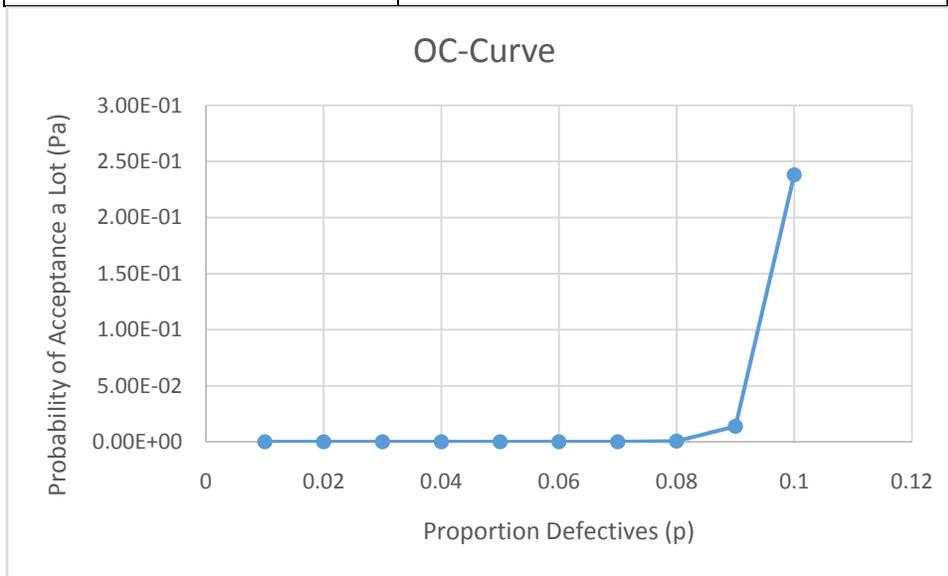
Now, assuming sample size i.e. **n = 400**, acceptance number **c = (P (default)\*400) = 2**

i.e if the no. of defectives is less than or equal to c , the lot is accepted , but if it is more than c, the lot is rejected.

Assume the percentage of defectives (p) in a lot as 1%, 2%, 3%, 4%, 5%, 6%, 7%, 8%, 9% and 10%. Then, calculate probability of acceptance ( $P_a$ ) for each percentage of defectives (p) using the Poisson's distribution equation as mentioned above.



P	Pa
0.01	3.57287E-15
0.02	1.58887E-13
0.03	6.90197E-12
0.04	2.91096E-10
0.05	1.18162E-08
0.06	4.55515E-07
0.07	1.63176E-05
0.08	0.000522258
0.09	0.013753968
0.1	0.238103306



The expected quality of the lots after the applications of sampling inspection is called the average outgoing quality (AOQ).

It is calculated using the following formula:  $AQO = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}}$  i.e. Taking  $N$  as the lot size,  $n$  as

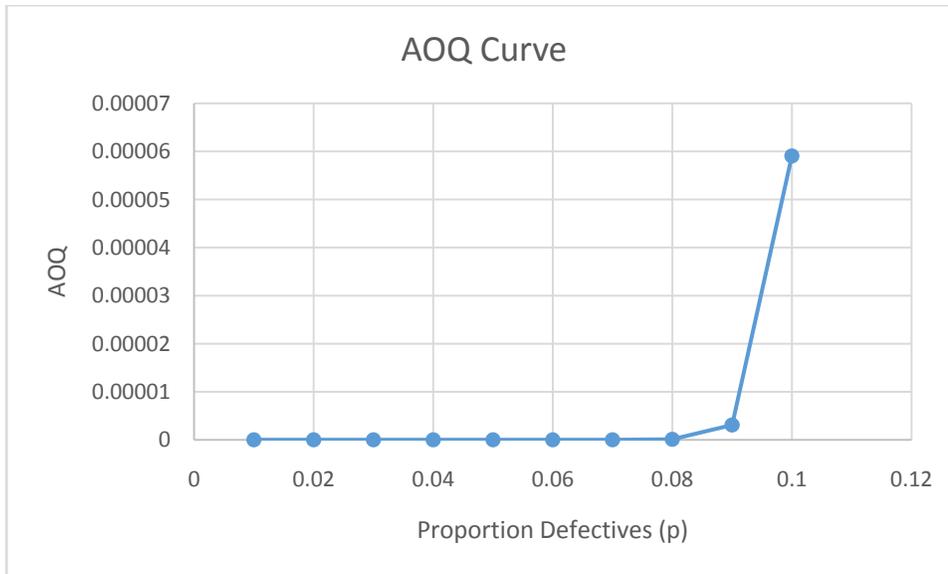


the sample size,  $p$  as the incoming lot quality,  $p_a$  as the probability of accepting the lot using the given sample plan, the average outgoing quality is given by

$$AOQ = \frac{N-n}{N.n} \cdot p \cdot p_a$$

The probabilities of accepting the lot and the AOQs corresponding of different quality level are given in the following table:

P	Pa	AOQ
0.01	3.57287E-15	8.86072E-20
0.02	1.58887E-13	7.8808E-18
0.03	6.90197E-12	5.13507E-16
0.04	2.91096E-10	2.88767E-14
0.05	1.18162E-08	1.46521E-12
0.06	4.55515E-07	6.77806E-11
0.07	1.63176E-05	2.83274E-09
0.08	0.000522258	1.03616E-07
0.09	0.013753968	3.06989E-06
0.1	0.238103306	5.90496E-05



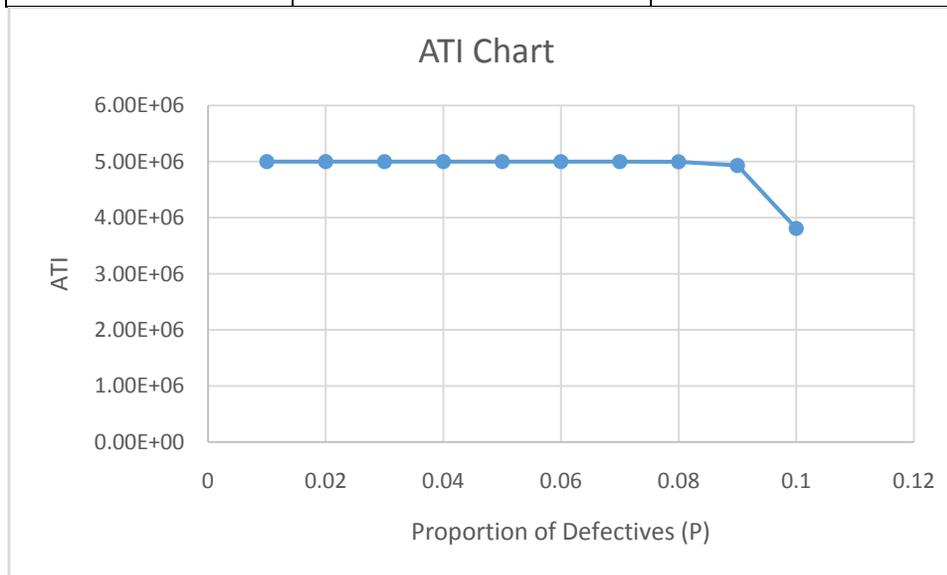
We now construct the AOQ curve by taking the quality level (proportion defective) on the X – axis and the corresponding AOQ values on the Y – axis.

The average number of units inspected per lot under the rectifying sampling plan is called the **average total inspection(ATI)**. Therefore, we can compute the ATI for a single sampling plan as follows:

$$\begin{aligned}
 \text{ATI} &= \text{number of units inspected per lot} \\
 &= \sum (\text{inspected no. of units} * \text{probability of taking the decision}) \\
 &= n * P_a + N * (1 - P_a) \\
 \text{ATI} &= n + [1 - P_a(p)] (N - n)
 \end{aligned}$$

The probabilities of accepting the lot and the ATIs corresponding to different quality levels are given in the following table:

P	Pa	ATI
0.01	3.57E-15	5.00E+06
0.02	1.58887E-13	5.00E+06
0.03	6.90197E-12	5.00E+06
0.04	2.91096E-10	5.00E+06
0.05	1.18162E-08	5.00E+06
0.06	4.55515E-07	5.00E+06
0.07	1.63176E-05	5.00E+06
0.08	0.000522258	5.00E+06
0.09	0.013753968	5.00E+06
0.1	0.238103306	5.00E+06



We now construct the ATI curve by taking the quality level (proportional defective) on the X – axis and the corresponding ATI values on the Y – axis.



The **average sample number (ASN)** is the expected number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under the acceptance sampling plan.

In acceptance single sampling plan, the decision about the acceptance or rejection of a lot is taken on the basis of a single sample that has been inspected. Therefore, the ASN in a single sampling plan is simply the sample size  $n$ . It means that ASN is constant in a single sampling plan. Therefore,  $ASN = n$ .

The curve drawn between the ASN and the lot quality ( $p$ ) is known as the ASN curve. The ASN curve for a single sampling plan is a straight line.

## 5. CONCLUSION

From the detailed analysis is done above, we may derive the following inferences:

- We see that the OC–curve has a negative slope, i.e. as the proportion of defectives increases, the probability of accepting the lot the decreases.
- If LTPD is taken to be 10%, OC curve shows that  $P_A$  is very low. Even if LTPD is lower than 10%,  $P_a$  does not rise over 0.24, which implies consumer risk of receiving a defective product is very low, especially in the automobile sector.
- The AOQ curve plotted shows that as the proportion of defectives increases, the average quality of the outgoing lot decreases.
- In the ATI curve the average number of units inspected per lot remain almost the same for proportion of defectives ranging from 2% - 10%.

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