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FUNDAMENTAL CONCEPTS OF NOTATIONS OF SEQUENCE SPACES AND FUZZY SEQUENCE SPACES

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ABSTRACT

The space of all entire functions is an important subject which unifies function theory and complex analysis. This theory is extended to matrix transformations involving entire sequences. In several branches of analysis, the study of sequence spaces occupies a very prominent position. The theory of sequence spaces is a part of functional analysis, motivated by problems in Fourier series, power series and systems of equations with infinitely many variables. Apart from this, it is a powerful tool for obtaining positive results concerning Schauder bases and their associated types. Also it has made remarkable advances in recent times in enveloping summability theory via unified techniques effecting transformations from one sequence space into another. Therefore this study is made on this subject matter.

Keyword: Fuzzy sequences, summability, power series, matrix transformation

1. INTRODUCTION

The theory of sequence spaces, topologized in a variety of ways has been developed in considerable detail. General class of sequence space was introduced [1].

The space of entire functions was first formulated. Subsequently, he contributed his results on this space in a series of papers. It is investigated properties of the Hilbert space of entire functions. Certain subspaces of the space of entire functions were propounded [2].

The most general linear operator acting between sequence spaces is actually determined by an infinite matrix. In 1911, the celebrated German Mathematician Otto Toeplitz determined necessary and sufficient conditions on an infinite matrix which maps convergent sequences into convergent sequences [3].

The study on matrix transformations was made by many authors. The entire method of summation was investigated by H. I Brown and further properties of entire method of summation were studied [4].

Today a lot of research is going on in Orlicz sequence spaces, difference sequence spaces and statistically convergent sequence spaces. This paper focuses on entire sequence spaces.

2. NOTATIONS IN SEQUENCE SPACES

The following notations are used throughout this work [5].

N = The set of all natural numbers.

R = The set of all real numbers.

C = The set of all complex numbers.

K = The field of real or complex numbers equipped with the metric topology generated by modulus on it.

ω = The family of all sequences (x_n) with $x_n \in K$, $n \geq 1$.

Let Φ be the sequence space of finitely non zero sequences from K defined by

$\Phi = \{x \in \omega : x_n = 0 \text{ for all } n \geq n_0 \text{ for some } n_0 \in N\}$.

3. FUNDAMENTALS OF SEQUENCE SPACES

Definition 3.1. Let X be a sequence space then X is solid, if $y_n \in X$ whenever $x \in \omega$ with $|y_n| \leq |x_n|$, $n = 1, 2, 3, \ldots$ for some $x_n \in X$. Some authors refer a solid space as a normal space. X is symmetric, if when $x_n \in X$ then y_n is in X and when the coordinates of y are those of x but in different order (that is, $y_n = x\pi(n)$, $n = 1, 2, \ldots$) for some permutation π on N [6].

For a subsequence J of N and a sequence space X, define XJ = {x_n∈ω : there is a y_n∈ X with x_n = y_{mn} for all mn∈ J} and call XJ as the step-space. If xJ∈ XJ then the canonical preimage of xJ is the sequence ˜xJ which agrees with xJ on the indices in J and is zero elsewhere. The canonical preimage of XJ is the space X˜ J containing canonical preimages of the elements xJ . X is monotone, if X contains the canonical preimages of all its step spaces.

Definition 3.2. A directed set is a partially ordered set with the additional property that for each x, y, there exists z with $z \geq x$ and $z \geq y$. For example, R with the usual order is a directed set.

A net is a function defined on some directed set. For example, a sequence is a net defined on the positive integers.

A net on real numbers is a function $x_\alpha : D \rightarrow R$ where D is some directed set. It is denoted by (X_α : D).

A Cauchy net is a net (X_α : D) such that, for any neighbourhood G of 0, there exists δ_0∈ D such that $\delta \geq \delta_0$, $\delta' \geq \delta_0$ imply $X_\delta - X_{\delta'} \in G$. Cauchy sequence is a Cauchy net.

Definition 3.3. The linear space X consisting of the continuos linear functionals on X may bear various topologies. The weak topology σ(X ′ , X) is the topology of pointwise convergence on X. The Mackey topology τ (X ′ , X) is the topology of uniform convergence on convex, circled, weakly compact subsets of X. The strong topology β(X ′ , X) is the topology of uniform convergence on bounded subsets of X.

Let X be a locally convex space with the topology $\mathfrak{I}$.

Definition 3.4. If each closed and bounded subset of X is compact then X is said to be Semi-Montel.

Definition 3.5. For a sequence space X if X = Xςς then X is called a ς- perfect space (ς = α, β, γ). In particular a_n α-perfect space is called a Kothe space or just a perfect space.

4. NOTATIONS IN FUZZY SEQUENCE SPACES

Definition 4.1. Let X be a non empty set. A function $\mu : X \to [0, 1]$ is called a fuzzy set on X. For any $x \in X$, a membership $\mu(x)$ is called the membership grade of x [7].

Definition 4.2. A fuzzy number is a fuzzy set on the real axis. That is, a mapping $u : R \to [0, 1]$ which satisfies the following four conditions.

(i) u is normal, means that there exists an xo ∈ R such that u(xo) = 1.

(ii) u is fuzzy convex, that is, $u[\lambda x + (1 - \lambda)y)] \geq \min\{u(x), u(y)\}$ for all x, y ∈ R and for all λ ∈ [0, 1].

(iii) u is upper semi continues.

We denote the set of all fuzzy numbers on R by E ′ and called it as the space of fuzzy numbers. λ-level set $[u]_\lambda$ of u ∈ E ′ is defined by

$$[u]_\lambda = \begin{cases} \{t \in \mathbb{R} : u(t) \geq \lambda\} & \text{if } (0 < \lambda \leq 1) \\ \overline{\{t \in \mathbb{R} : u(t) > \lambda\}} & \text{if } (\lambda = 0). \end{cases}$$

The set $[u]_\lambda$ is a closed, bounded and non-empty interval for each λ ∈ [0, 1], which is defined by $[u]_\lambda = [u -(\lambda), u+(\lambda)]$. Since each r ∈ R can be regarded as a fuzzy number r defined by

$$\overline{r} = \begin{cases} 1 & \text{if } (x = r) \\ 0 & \text{if } (x \neq r) \end{cases}$$

Representation Theorem: Let $[u]_\lambda = [u -(\lambda), u+(\lambda)]$ for u ∈ E ′ and for each λ ∈ [0, 1]. Then the following statements hold [8].

(i) $u^-(\lambda)$ is a bounded and non-decreasing left continuous function on [0, 1].

(ii) $u^+(\lambda)$ is a bounded and non-increasing left continuous function on [0, 1].

(iii) The functions $u^-(\lambda)$ and $u^+(\lambda)$ are right continuous at the point $\lambda = 0$.

(iv) $u^-(1) \leq u^+(1)$.

Conversely, if the pair of functions α and β satisfies the conditions (i)-(iv) then there exists unique $u \in E'$ such that $[u]_\lambda = [\alpha(\lambda), \beta(\lambda)]$ for each $\lambda \in [0, 1]$ [9]. The fuzzy number u corresponding to the pair of functions α and β is defined by

$u : R \to [0, 1], u(x) = \sup\{\lambda : \alpha(\lambda) \leq x \leq \beta(\lambda)\}$.

5. FUNDAMENTALS OF FUZZY SEQUENCE SPACES

Definition 5.1. A sequence $u = (u_k)$ of fuzzy numbers is a function u from the set N into the set E'. The fuzzy number u_k denotes the value of the function at $k \in N$ and it is called as the k th term of the sequence [10].

Lemma 5.1. The following statements hold.

(i) $D(uv, 0) \leq D(u, 0)D(v, 0)$ for all $u, v \in E'$.

(ii) If $u_k \to u$ as $k \to \infty$ then $D(u_k, 0) \to D(u, 0)$ as $k \to \infty$, where $(u_k) \in w(F)$

Definition 5.2. A sequence $(u_k) \in w(F)$ is called convergent with limit $u \in E'$ if and only if for every $\varrho > 0$ there exists an $n_0 = n_0(\varrho) \in N$ such that $D(u_k, u) < \varrho$ for all $k \geq n_0$.

Definition 5.3. A sequence $(u_k) \in w(F)$ is called Cauchy sequence of fuzzy numbers if and only if for every $\varrho > 0$ there exists an $n_0 = n_0(\varrho) \in N$ such that $D(u_i , u_j) < \varrho$ for all $i, j \geq n_0$.

Definition 5.4. A sequence $(u_k) \in w(F)$ is called bounded if and only if the set of all fuzzy numbers which contains the range of the sequence (u_k), is a bounded set. That is, to say that a sequence $(u_k) \in w(F)$ is said to be bounded if and only if there exist two fuzzy numbers m and M such that $m \leq u_k \leq M$ for all $k \in N$

Definition 5.5. If (u_k) is a sequence of fuzzy numbers and (n_k) is an increasing sequence of positive integers, then the sequence (u_{nk}) is called a subsequence of (u_k).

Proposition 5.1. If (u_k) is a sequence of fuzzy numbers, then the following statements hold:

(i) If a sequence (u_k) is convergent, then it has unique limit.

(ii) Any convergent sequence (u_k) of fuzzy numbers is bounded.

(iii) If (u_k) converges, then any subsequence of (u_k) converges to the same point.

Definition 5.6. A sequence space $E(F) \subset \omega(F)$ is said to be solid, if $D(v_k, 0) \leq D(u_k, 0)$ for all $k \in N$ implies $(v_k) \in E(F)$ whenever $(u_k) \in E(F)$, where

$$\overline{0}(t) = \begin{cases} 1, & t = (0, 0, \ldots, 0) \\ 0, & \text{otherwise.} \end{cases}$$

Definition 5.7. A sequence space $E(F) \subset \omega(F)$ is said to be symmetric, if $(u_{\pi(n)}) \in E(F)$ whenever $(u_k) \in E(F)$, where π is a permutation on N.

Definition 5.8. Let $(u_k) \in w(F)$. Then the expression

$$\sum_{k=0}^{\infty} u_k$$

is called as series of fuzzy numbers.

Let us denote $s_n = \sum_{k=0}^{n} u_k \;\; for\ all\ n \in \mathbb{N}$

If the sequence (S_n) converges to a fuzzy number u then we say that the series $\sum_{k=0}^{\infty} u_k$ has the sum u and we write $\sum_{k=0}^{\infty} u_k = u$.

That is,

$$\sum_{k=0}^{n} u_k^-(\lambda) \to u^-(\lambda) \ \& \ \sum_{k=0}^{n} u_k^+(\lambda) \to u^+(\lambda)$$

uniformly in [0, 1] as n → ∞.

We say otherwise, the series of fuzzy numbers diverges.

Additionally, if the sequence (S_n) is bounded then we say that the series $\sum_{k=0}^{\infty} u_k$ of fuzzy numbers is bounded.

By $c_s(F)$ and $b_s(F)$ we denote the set of all convergent series of fuzzy numbers and bounded series of fuzzy numbers, respectively

Consider the series $\sum_{k=0}^{\infty} u_k$ with

$$u_k(t) = \begin{cases} 1-(K+1)^2, & 0 \le t \le \dfrac{1}{(k+1)^2} \ for\ all\ k \in \mathbb{N} \\ 0 & otherwise \end{cases}$$

This series converges to u, where

$$u(t) = \begin{cases} 1-\frac{6}{\pi^2}t, & 0 \le t \le \frac{\pi^2}{6} \\ 0 & \text{otherwise.} \end{cases}$$

Definition 5.9. Let S_1 and S_2 be two sequence spaces and A = (a_{nk}) be an infinite matrix of real or complex numbers. Then the matrix A defines a transformation from S_1 into S_2, if for every sequence x = (x_k) ∈ S_1, the sequence.

$A_x = ((A_x)_n)$ exists and is in S_2, where $(Ax)_n = \sum_{k=1}^{\infty} a_{nk} x_k$

For a sequence space S, the matrix domain S_A of an infinite matrix A is defined by

$S_A = \{x = (x_k) \in w : A_x \in S\}$.

6. CONCLUSION

It is concluded that among the various paradigmatic changes in science in this century, one such change concern the concept of uncertainty. According to modern view, uncertainity is considered essential to science, it is not only an unavoidable plague, but it has in fact a great utility. Many research investigations by scientists and engineers all over the world have been made in the theory and applications of the subject. Research on the theory of fuzzy sets has been growing steadily since the inception of the theory in the mid-1960s. Research on a broad variety of application has also been very active and impressive. This motivate us to study the fuzzy sequence spaces. In recent years, the fuzzy theory has emerged as an active area of research in many branches of mathematics and engineering. The theory of fuzzy numbers is not only the foundation of fuzzy analysis but it also has important applications in fuzzy optimization and fuzzy decision making etc.

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