An overview of linear and nonlinear penetrative convection in fluid and porous layers with regulated heat flux on the boundaries

Nagarathna N
Associate professor
Department of Mathematics
Maharani's Science College for Women,
Bengaluru, Karnataka, India.

Abstract

This review embodies the theoretical investigation of three types of problems viz., a general model for the Technique of Reconstitution. Penetrative convection in fluid and porous layers under different constraints like, rotation, salinity gradient, nonlinear temperature profile etc., and thermosolutal instability in homogenous and heterogeneous layers in the presence of coupled molecular diffusion. The aim of the present investigation is to provide the qualitative as well as the quantitative features of the phenomena e.g., about the form as the flow pattern, the size of the convective cell, the temperature, salinity and velocity distributions, the formation of horizontally long convection cells, occurrence of subcritical motions, the construction of the evolution equation, the amplitude etc.In type I, in order to elucidate the properties of reconstituted equations by applying the technique to a highly complicated system of nonlinear equations, a general model consisting of coupled three nonlinear differential equations are considered. In Type II, five models are discussed where the boundaries have fixed-heat and salt flux conditions. In Type III, the boundaries are of free-slip type.

Introduction

In recent years, the study of the phenomenon of the convective process in a horizontal fluid/porous layer has received remarkable attention owing to its very wide applications in science, engineering and industrial areas. Convection can be the dominant mode of heat and mass transport in many processes that involve the freezing and melting of the storage material. Natural convection is an omnipresent transport phenomenon in saturated porous geological structures. In fact, the study of convection is of most importance in geophysical, astrophysical and heat transfer problems. For example, the extraction of energy from geothermal sources is the most promising one among the other methods and it is believed that the fluid in these reservoirs is highly permeable and consists of multi-components rather than a single component. Therefore, buoyancy driven convection in a porous medium with water as the working fluid is an important mechanism of energy transport. In fact, the key feature of a major geothermal system while on the land or beneath the sea floor is a high intrinsic heat transport. In fact, the local thermal conditions and the physical properties of media are directly of great importance on the characteristics of the heat and mass transfer in such real configurations. Moreover, the nature of the fluid flow (2D, 3D, pattern and range) is drastically dependent on the complexity of geophysical sites, i.e., the geometry, heterogeneity and anisotropy of the domains. Fluid motions induced by free convection have tangible effects in geothermal areas, on the diffusion of pollutants or on the mineral diagenesis processes.

Technically, the phenomenon is important as it may occur in porous insulation of buildings thereby increasing the loss of heat. In the stellar atmosphere also, certain heavenly bodies may be considered to be porous material and the study has relevance to that also. Also, convection in planetary cores and stellar interiors often occurs in the presence of strong rotational and magnetic constraints. Over the past four decades, there has been an increasing concern about soil and water contamination from industrial and agricultural chemicals. In such cases, thermal and chemical interactions between a rotating porous layer and an overlying fluid layer can be considered. Such a study has many engineering and environmental applications also.

The study of convection in a fluid saturated porous layer is also of interest since it provides a convenient means for experimentally determining the nonlinear effects such as, the preferred cell pattern, heat transport etc. In the case of Rayleigh-B’enard Convection, it is necessary to consider a thin fluid layer to detect these phenomena whereas in porous media, the depth of the fluid can be greatly increased since the frictional force is much larger. The study of porous convection has attracted the attention of considerable research workers because of its natural occurrence and its intrinsic importance in many industrial problems, particularly in petroleum exploration, chemical and nuclear industries. The mechanism of transfer of heat from the deep interior of the earth to a small depth in the geophysical region is of vital importance. These studies also help in power generation. More specifically, the results of the study of natural convection in a porous medium are useful in nuclear industries in the evaluation of the capability of heat removal from a hypothetical accident in a nuclear reactor.

As the fluid in the aquifers consists of multi-components rather than just a single component, there exists two sources of buoyancy. Further, multi-component onset of convection is important in many naturally occurring phenomena and technological processes. Examples include Convection in stars, dynamics within the earth's core, oceanography, solar ponds, coating/drying processes and crystallization/solidification.

The Coriolis force caused by earth's rotation plays a significant role in the determination of the qualitative and quantitative features of the system. The effect of geomagnetic field and earth's rotation on the stability of geophysical flows is of great interest to geophysicists. While studying the stability of earth's core, the role of magnetic field becomes important, where the earth's mantle consisting of molten conducting fluid, behaves like a porous medium and can become convectively unstable as a result of differential diffusion. The reason for the occurrence of this phenomenon is that the stabilizing effect of one component is reduced by diffusion in the presence of a magnetic field, thereby releasing the potential energy of the unstable component. In geophysical problems, the effect of earth's rotation is considerable and distorts the boundaries of a hexagonal convective cell in a fluid / porous layer and this distortion plays an important role in the process of extraction of energy.

Further, the understanding of the flow phenomenon in packed beds is of considerable practical importance especially in the interpretation of chemical reactor performance where hydrodynamic dispersion and molecular diffusion play important roles in mixing process. The coupled molecular diffusion phenomenon is also of interest in many problems such as

i) the movement of fertilizers in the oil and leaching of salts from soil in agriculture

ii) radio-active and reclaimed sewage waste disposal into aquifers

iii) the transition zone between salt water and fresh water and

iv) blood flow through a capillary etc.

Convective instabilities arise as a result of unstable equilibrium in a region of the fluid / porous layer. Accordingly, when the region of unstable equilibrium is bounded by the fluid that is in stable equilibrium, in most of the cases the associated convective motion penetrates into the neighbouring regions of stable equilibrium. The cause for the penetration is (i) the velocities and (ii) the non-vanishing of the tangential stresses of the perturbed motion at the region of static stability.

Penetrative convection arises in many geophysical and astrophysical situations. In the atmosphere a statically unstable layer is always surrounded by a stable region. For example, in the atmosphere, solar radiations can heat air near the surface of the earth or ocean and generate a gravitationally unstable layer beneath a stable stratified environment. Another example of interest is a low-level inversion or stratosphere in the case of deep convection.The mechanism is interesting in the sense that, when convective motion occurs in the lower layer, it mixes with the overlying stable layer and thus convective motions penetrate into the stable fluid. The reciprocal situation of convection penetrating downwards from above can occur in lakes and oceans although in the ocean, upper mixed layer is typically formed by turbulence generated by surface wind. In other words, convective circulations in the well-mixed surface layer penetrate into the stable thermocline region. These examples of penetrative convection are principally unsteady and transient.

Statistically, stationary penetrative convection may occur in stars, where large changes in the mean-free path of photons cause large changes in the diffusion of heat with temperature. As a result, convective motions occur.

Another important example of stationary penetrative convection is that of convection in water of temperature near 4°C. In the case of water driven by buoyancy force through a porous structure, the flow pattern is influenced dramatically by the occurrence of a density maximum at 3.98°C when the pressure is atmospheric.

In most of the laboratory experiments pertaining to the phenomenon of convection, the unstable layer is sandwiched between rigid boundaries,but the stellar convection zones are bounded by stably stratified regions. Hence, the steady penetrative convection across the interface between stable and unstable layers is of astrophysical importance. In fact, in the stably stratified photosphere solar granulations is observed and may excite the oscillations detected in the upper atmosphere.

Penetrative convection may also affect nuclear abundances. For example, the apparent shortage of lithium in the sun and other late-type stars may be due to the slow mixing of material into the stable radiative zone and penetration can no longer be ignored in the physics of stellar interiors.

A thorough understanding of geophysical, astrophysical and meteorological convection process requires a good knowledge of the qualitative and quantitative features of penetrative convection in a fluid /porous layer under the influence of external constraints like, rotation, salinity gradient, nonlinear temperature profile etc. Therefore, an attempt is made in this review to know the onset of penetrative convection in fluid and porous layers in the presence and absence of external constraints, to predict the formation of horizontally long convection cells, occurrence of subcritical motions, the behaviour of the vertical and horizontal structure of the velocity fields, the construction of evolution equation etc.

Convective phenomenon-a brief review

The concept of convection is associated with the process of heat transfer through the motion of liquids or gas. It is that process of heat transport in which there is a movement of the macro-particles of the liquid or gas.

Convective instability arises, whenever there is an imbalance between the viscous and the buoyancy forces. The first transition will be conduction to convection and thereafter the motion becomes super-critical and finally leads to turbulence if the associated parameter (Rayleigh number) is sufficiently high. There are basically four types of convection. The following diagram gives a clear picture of the different sub-sections associated with the convection phenomenon.

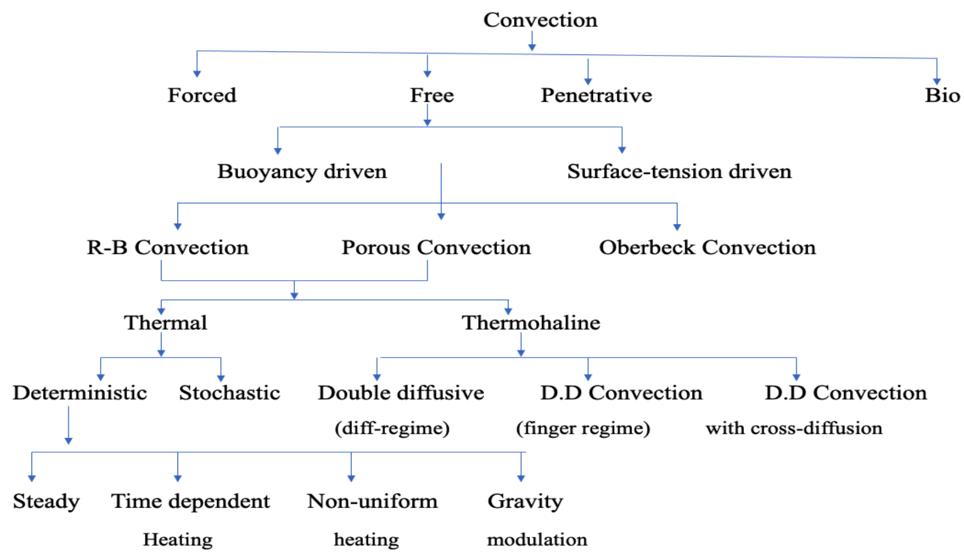

Figure 1: Various types of Convective phenomena

The study pertaining to this review is mainly concerned with penetrative convection in the free convection environment, induced by buoyancy forces.

Buoyancy driven convection is due to the density differences which result due to the temperature/concentration variations. Broadly speaking thermal convection is of two types

i) Overbeck convection

ii) Rayleigh-B'enard convection

In the first type, the configuration is such that the fluid layer is subjected to a temperature gradient normal to the direction of gravity, whereas in the second type the fluid will be subject to a temperature gradient parallel to the gravity but opposing it. Convection sets in when the destabilizing effect of the temperature difference across the layer overcomes the opposing forces (viz, viscosity and action of conductivity) and is characterized by a dimensionless parameter called "Rayleigh number".

The density difference in the case of buoyancy driven convection may arise due to the vertical gradient of salinity present in the fluid/porous layer. A fluid layer containing vertical gradients of both temperature and salinity is subject to several instabilities arising in industrial and geophysical problems.

The minimum requirements for the occurrence of double-diffusive convection are the following

i) The fluid must contain two or more components having different molecular diffusivities. It is the differential diffusion that produces the density differences required to drive the motion.

ii) The components must make opposing contributions to the vertical density gradient

(It is assumed throughout that the fluids are completely miscible, so that surface-tension effects do not arise).

More generally, we can distinguish four cases

Table 1: Four different types of cases

Case	$\frac{dT}{dz}$	$\frac{dc}{dz}$	$\frac{d\rho}{dz}$
1	+	-	-
2(a)	+	+	-
2(b)	+	+	+
3(a)	-	-	-
3(b)	-	-	+
4	-	+	+

Case 1:

It is gravitationally stable; both the gradients are stabilizing; convection is not possible since the density decreases upward.

Case 2(a) and 3(a):

In these cases, it is possible to have convection even though they are gravitationally stable.

Cases 4, 2(b) and 3(b):

These are gravitationally unstable and correspond to Rayleigh B'enard convection.

Cases 2(a) and 3(a) are interesting, when both salinity and temperature increase upwards (i.e., stabilizing temperature gradient and destabilizing salinity gradient), the layer may become unstable although the density profile indicates stability. The driving mechanism associated with this type of instability strongly depends on the different diffusive properties of heat and salt. The phenomenon of salt fountain which occurs when hot salty water lies above cold fresh water was discovered by Stommel, Arons and Blanchard (1956). Since then, considerable interest has been developed in this class of hydrodynamic instabilities. In general, they come under "multi-component convection" or "double-diffusive convection".

The experiments of Stern (1960) confirmed "salt-finger instability" where thin fingers of up and down going fluid were observed. The physical mechanism for this can be explained as follows

When a stably stratified horizontal fluid layer is subjected to a destabilizing salinity gradient, a parcel of fluid perturbed downwards loses its stabilizing temperature excess so much faster than its destabilizing salinity excess due to the difference in diffusivities of heat and salt, that it can continue to move downwards.

Thus, the setting up of convection will depend on the nature of the solute gradient (i.e., stabilizing or destabilizing). Thus, a stratified layer of a single component fluid in a porous medium is stable if the density decreases upwards whereas a layer consisting of more than two components (multi- component system) which can diffuse relative to each other may be dynamically unstable. Thus, the resulting buoyancy force may tend to increase or reverse the direction of the displacement of the particle from its original position and thus cause monotonic instability or over stability according as the temperature gradient is stabilizing and the salinity gradient destabilizing or vice versa.

The case of a stabilizing vertical solute gradient in a fluid saturated porous layer can serve to inhibit the onset of convection when the fluid is heated from below, for the Darcy's resistance together with the potential energy released by the horizontal temperature gradient is balanced in the absence of inertia and local acceleration. The larger the vertical solute gradient, the less the potential energy released for a given horizontal temperature gradient to balance with the Darcy's resistance. That is convection may set up at a thermal Rayleigh number considerably much less than that predicted by the linear theory. The apparent reason for such a finite amplitude subcritical instability is that motion of the fluid may distort the solute profile, so that away from the boundaries the stabilizing solute gradient is reduced and the destabilizing thermal gradient which is less affected by the motion because of the thermal diffusivity, may cause convection more easily, resulting in oscillatory instability and finite amplitude motions. However, subcritical instability of this type (oscillatory type) is not possible when the solute gradient is destabilizing.

The study of convection in a fluid/porous layer in the absence of penetration has received considerable attention in the past few decades and Copious literature on this is now available (Chandrasekhar (1961); Nield (1968); Rudraiah and Srimani (1976); Turner (1979); Rudraiah and Srimani (1980): Srimani 1981(hereafter referred to as I and II); Rudraiah, Srimani and Friedrich (1982); Nield & Bejan (1991); Srimani (1990); Srimani (1991); Srimani and Anamika (1991); Srimani and Sudhakar (1992); Koschmieder (1993); Srimani and Sudhakar (1996); Chevalier et al (1999); Sezai and Mohamad (1999); Payne and Song (2000); Srimani and Nagarathna (2000) Aurnou and Olson (2001); Westerburg and Busse (2001).

A careful survey of the literature pertaining to penetrative convection in porous media shows that very sparse literature is available in this direction. Although penetrative convection occurs abundant in nature, surprisingly very sparse literature is available. It is presumably because of the mathematical difficulties involved therein.

Therefore, in this section, only the works pertaining to penetrative convection is reviewed in brief with the motive of paving the way for further work.

In this review, three types of problems are investigated viz.

I. Model for the study of the Technique of Reconstitution

II. Linear and nonlinear penetrative convection under different constraints

III. Thermosolutal stability caused by coupled molecular diffusion

Review-Type I

It is observed that, in many physical problems the solution is dominated by a particular structure, and it is possible to derive a differential equation which is capable of describing the spatial and/or temporal evolution of this dominant structure. Such an evolution has its own limitations and is valid only for a restricted range of the parameters. But the Technique of Reconstitution provides a rationale for systematically making corrections to this first approximation.

Roberts (1985) extended the method proposed by Spiegel (1981), for correcting such evolution equation by adding extra terms which bring new physics into the equation. He has illustrated the technique by considering a simple pair of coupled nonlinear differential equations and has shown that the advantage of the procedure of reconstitution is that the physical processes which previously could only slightly modify the leading-order solution can now interact with the dominant dynamics of the leading-order evolution equation.

Review-Type II

Penetrative convection in fluid layers

The earliest theoretical treatment of penetrative convection is the work of Veronis (1963). He has considered a simple model of an infinite layer of fluid of finite depth with a linear temperature profile across it. The density-temperature relationship is assumed to be quadratic, and the maximum density occurs somewhere in the interior.

He has performed both linear and nonlinear stability analyses of the ice-water problem. His results predict that when instability occurs in the unstable layer it penetrates into the stable layer as well. Multi-cells are predicted. He has found the critical Rayleigh number to be dependent on the position of maximum density in the layer. His limited nonlinear analysis however reveals that the layer could become unstable to finite amplitude perturbations at $R < R_c$.

Experimental investigations pertaining to ice-water problem are conducted by Furumoto and Rooth (1961), Townsend (1964) and Myrup et al (1970). However, information pertaining to general penetrative convection in deep sea, ocean etc. are available in the works of Warner and Telford (1967), Deardorff et al (1969), Whitehead and Chen (1970), Tankin and Farhadieh (1971),Spigel (1972), Willis and Deardorff (1974), Adrian (1975), Farmer (1975), Turner (1979) and Walden and Ahlers (1981), reviews the relevance of penetrative convection to a variety of natural phenomena.

Penetrative convection motion due to the non-vanishing of the tangential stresses at the boundary of the region of inviscid instability was investigated by Taylor (1923). Encouraged by the Taylor's explanations of physical significance of the problem, Rintel (1967) has used an approximate method for finding the critical conditions for the stability of flow between counter rotating cylinders and has compared with the experimental results. He has extended the method to study (i) the flow between counter-rotating concentric cylinders (ii) the classical B'enard problem arising in a horizontal fluid layer with constant unstable density gradient and penetrating into a layer of fluid with constant stable density gradient situated above the unstable layer. He has concluded that the classical rigid-free boundary solution corresponds to the limiting case of the infinite stability on the top of the stable layer.

Faller (1968) has commented on the work of Rintel (1967) regarding the analogy drawn in the case of penetrative convective instability in the classical experiment of counter-rotating cylinders and the thermal convective instability with rigid boundaries when the temperature is a parabolic function of height. But Rintel (1968) has answered the comments by saying that the adjoint systems with free boundaries have the same critical values.

Musman (1968) predicted the transition to finite amplitude convection through his numerical calculations for the ice-water system confined between the free upper and lower surfaces of infinite horizontal extent. He employed mean field approximation for solving the nonlinear governing equations. He confirmed the results of Veronis (1963) on the subcritical instability for penetrative convection. Musman also found that for water near its freezing point with the lower boundary held at 0°C, the upper boundary was not dynamically relevant to the system of the stable portion if the layer was sufficiently thick.

The works of Debler (1966) and Watson (1968) are relevant to the case of continuous parabolic profile of temperature, and it is comparison with rotating cylinder experiments. Deardorff et al (1968) have given an excellent experimental study that have a rather direct application to the finite amplitude time-dependent geophysical problems. But the stability of thermal convection for a statically unstable layer surmounted by a statical stable layer has been attacked by Faller and Kaylor (1970) by direct numerical integration of the linearized equations of motion and heat conduction. They have considered two types of vertical temperature variation: (i) a piecewise linear distribution and (ii) a piecewise-parabolic distribution. For the first case, their initial computation showed a very large penetration of the cells into the stable layer when compared to the approximate method of Rintel(1967).However, for the second case, the penetration was more than twice the penetration for the corresponding problem with counter-rotating cylinders (Harris and Reid 1964).

The ratio of the vertical gradient of temperature in the upper layer to the corresponding gradient in the lower layer was specified by a parameter win both the cases. Their numerical procedure provides the numerical values of the critical Rayleigh number, the most unstable wavelengths and the depths of penetration of the convective cells into the stable region for several values ofΨ.

Sun et al (1969) has made a theoretical and an experimental study of thermal instability of a horizontal layer of liquid with maximum density by using a density-temperature relationship that has a wider range of applicability. They have considered two types of hydrodynamic boundary conditions: rigid-rigid and free-free, because of their equal importance. The Rayleigh number is found to be dependent upon two parameters. Their study pertains to the onset of convection and their theoretical and experimental results are in good agreement. Their experimental study consists of the measurements of the melting rate of a block of ice with melting from both below and above.

The ice-water experiment provides an example of an essentially nonlinear fluid-dynamical problem whose behaviour can be well understood in simple terms. Moore and Weiss (1973) have conducted a series of numerical experiments for steady two-dimensional penetrative convection between free boundaries in great detail. They have also explored time-dependent behaviour together with the effect ofdifferent types of boundary conditions. Their results suggest the existence of subcritical motions. At high Rayleigh numbers, they have found that resonant coupling between convection and gravitational modes in the stable layer excites finite amplitude oscillations. They have found a good agreement between the numerical and experimental results. They have computed Nusselt number as a function of the Rayleigh number.

The two examples of penetrative convection viz., (i) the growth of a turbulent atmospheric boundary layer during early morning heating in the absence of wind and (ii) the deepening of the surface in a large deep power station cooling pond, are of great importance in the present-day environmental problems. The dispersal of pollutants released into the atmospheric depends on the rate of growth of the boundary layer. In a similar way the penetrative convection in a cooling pond also affects the heat loss from the warm inflow and hence, the power station efficiency. A major difference between the two examples is the role of molecular diffusion and, Peclet number is the governing parameter, because (Peclet number $p_e = I_s V_s / \kappa$), where$I_s$and $V_s$are the turbulent length and velocity scales and κ is the molecular diffusivity.

Although Gebhart and Mollemdorf (1977) investigated thermohaline convection in fluid mixtures that possess density maxima, neither their analysis nor their results are directly applicable to instabilities arising in infinite fluid layers.

Merker et al (1979), studied the onset of convection in a horizontal water layer with maximum density effects by assuming a fifth-order polynomial for the density-temperature relation. He concluded that the critical Rayleigh numbers computed, with a simple parabolic relation are about 10% larger.

The experimental results of Walden and Ahlers (1981) revealed hysteretic transition near the critical Rayleigh number for liquid helium which has a density maximum just above the super fluid transition temperature.

Roberts (1985) has made a detailed analytical study of mildly penetrative convection in a horizontal layer. The model considered is ice-water convection problem. Both linear and nonlinear analyses are made through the remarkable analytic simplification that emerges from specifying the heat flux on the boundaries rather than fixing the temperature. These boundary conditions are meaningful in geographical applications, because there is no guarantee that the commonly used boundary condition of fixed temperature is appropriate. Evidently, when the sun heats the air next to the ocean or ground, a fixed heat flux boundary condition is much more appropriate.

Another example is Mantle convection that occurs between poorly conducting boundaries and can be modelled by fixed-heat flux boundary conditions (Chapman et.al. 1980). Roberts has made the assumption of long horizontal scales of motion which restricts to consider the stably stratified layers that are relatively, shallow. thinner than about 65% of the thickness of the unstable layer. Density-temperature relationship is considered to be quadratic. The result of the linear stability analyses predict that long horizontal scales are preferred when the convection is mildly penetrative i.e., the overlying layer of stable fluid is not deep. Using the technique of reconstitution, he has investigated some of the physical processes of the finite amplitude convection. He has also calculated estimates for the maximum extent of subcriticality at which finite-amplitude convection may occur.

All the studies cited above for penetrative convection was concerned with the Benard type of convection in which heat was the only diffusing component. It is well known (See II) that a fluid/porous layer with two or more diffusing components exhibit instabilities even when the total density of the fluid decreases upward.

Antar (1987) has made a numerical study of penetrative double-diffusive convection, i.e., he has extended the study of penetrative convection to fluids possessing two diffusive components. The layer is considered to be infinite horizontal extent and has a density maximum at the interior. For the solution of the eigenvalue problem, he developed a computer code, with an eight-order, variable-step, Runge-KuttaFehlberg initial value generator. A Newton-Raphson method was used for the iteration procedure and the orthogonalization procedure was implemented at each integration step. The density-temperature relationship is assumed to be quadratic in the above investigation. His results predict the regions of stability and instability to both steady and oscillatory modes and are confined to the positive quarter plane of the Rayleigh and solute Rayleigh number plane. He found that the decrease in the position of maximum density in the vertical direction leads to a stability range and also an increase in the regions of oscillatory instability in that quadrant. His important conclusion is that the extent of penetration of the convective motion into the stable region is diminished with the increase in solute concentration. Another important conclusion is that, dynamically similar motions exist with significant quantitative differences in the case of rigid-rigid and free-free boundary conditions.

Matthews (1988) has proposed a model for penetrative convection discussing the stability of an S-shaped cubic temperature profile maintained by internal heating. In his model, the unstable layer is sandwiched between the two stable fluid layers. Computations are done for free-free and rigid-rigid boundaries. His results predict that the bifurcation is supercritical. A thorough comparison of his results and those of Veronis (1963) are made and discussed. Through numerical integrations, he has examined the qualitative behaviour of the problem.

Previous theoretical and experimental investigations on thermal instability with maximum and minimum density effects are confined to Rayleigh problem only. However, for thin horizontal liquid layers with an upper free surface, the onset of convection can be induced by surface-tension gradients and buoyancy forces (Pearson 1958, Neild 1964, Kobayashi 1967). Motivated by this result Wu and Cheng (1976) investigated the onset of cellular convection driven by surface tension and buoyancy force in a horizontal thin liquid layer by considering the density inversion effect for water by using a cubic density-temperature relationship. The lower boundary is considered as a rigid and thermally conducting while at the upper surface Pearson's boundary conditions are imposed to facilitate the analysis. The liquid layer associated with the maximum density effect can become unstable regardless of whether the heating is from below or above. The temperature regime under the consideration is from 0°C to 30°C. Their study involves the thermal parameters λ_1 and λ_2 and three physical parameters viz., Biot number, Rayleigh number and Marangoni number. By using a numerical procedure, they have made a detailed study of the linear stability problem by studying the neutral stability curves for different ranges of the parameters and the relations connecting the physical parameters are clearly presented. Results are quite interesting.

Richard et al. (1981) considered a theoretical one-dimensional model of penetrative convection in a stable temperature stratification heated from below by assuming the partial derivations of temperature with respect to the height and time to be discontinuous at the interface. At a finite temperature gradient, the molecular diffusion, effects at low Peclet numbers are included and a numerical study is made to illustrate the relative contribution of molecular diffusion, interfacial turbulence and the 'filling' of the existing temperature stratification by the lower boundary heat flux. The results of this detailed numerical study are confirmed and verified by the available experimental data.

The work of Normand and Azouni (1992) bears a close analogy with those cited above even if the destabilizing mechanisms are different. They have provided a model for penetrative convection in which a stably stratified layer of fluid is bounded by two unstable layers. Water layer around its density maximum is considered and the quadratic temperature profile is maintained by internal heating.

Their linear stability analysis predicts that either stationary or oscillatory modes occur at the onset of instability depending on the values of the control parameter. They have made a detailed numerical study of the two kinds of stationary models that can be attributed to a crossover between the fundamental and the first excited mode as the parameter u varies. They have discussed their results in the light of the study made by Rasenat et al. (1989). They have shown that for $\mu = 4.67$, it prevents penetration, and the instability remains confined in the predominant upper layer.

Sudhakar (1993) has discussed double diffusive penetrative convection in a fluid layer by considering a nonlinear equation of state and a linear temperature profile. He has assumed both the boundaries to be stress-free and perfect conductors of heat and salt. The method employed is the modified power integral technique which is well documented in the works of Veronis (1963), I and Srimani and Sudhakar (1992). He has discussed the possible cases for the existence of single cell and multi-cells. He as shown that the penetration Rayleigh number assumes a local maximum at the value 2 of the depth parameter. His linear stability analysis gives the critical wavenumber, critical penetrative and Rayleigh numbers for different values of the depth parameter λ and solute Rayleigh number R_s. He has predicted the occurrence of subcritical motions. The heat transport curve for fixed R_s is interesting and is valid for a wider range of the Rayleigh number.

The author has also investigated the effect of magnetic field on the onset of penetrative convection in a fluid layer by considering a linear equation of state and an S-shaped cubic temperature profile. His model predicts the critical conditions and the bifurcation phenomenon. Fourier Transform technique and fourth order Runge-Kutta Gill method are employed. He has derived the amplitude equations resulting from the solvability conditions. He found that Prandtl as well as magnetic Prandtl numbers have a strong influence on the onset of penetrative convection. He as shown that the Counter- cells which are away from the main cell are weaker in magnitude and the effect of magnetic field is to make the system more stable in general.

Nishimura et al. (1995), have made a numerical study of Natural Convection of water near the density extremum for a wide range of Rayleigh numbers. Their calculations were conducted by deploying a mesh work of (62×52) staggered grid points in the (x-z) domain. The grid and time-step were varied for repeated calculations. They have considered different types of flows in their work.

Kwak et al. (1998) have made a detail study of convective cool-down of a contained fluid through its maximum density temperature. They have used time-dependent Navier-Stokes equations and have discussed the specific numeral techniques with regard to the problem. Their numerical calculations and the computed results for Nu are consistent with the earlier results.

Penetrative Convection in porous media

Sun et al. (1969) studied the thermal instability of a horizontal layer of liquid with maximum density by using a cubic density-temperature relationship. Sun et al. (1972) has extended the above work to include the effect of density maximum on the onset of convection in a porous medium.

Only linear stability analysis is carried out by using the empirical expression of Darcy's law. The modified Rayleigh number is found to be dependent on two parameters λ_1 and λ_2, respectively. To provide sufficient experimental data, Yen (1974) has made an experimental study of the effects of density inversion on free convective heat transfer in a porous medium. The experimental set up used is essentially the same as the one described in detail by Ten et al. (1972). He has found the critical Rayleigh number to be $4\pi^2$ for upper boundary at 4°C and 8°C in which case the effect of density inversion on the onset of convection gets eliminated. But the effect of density inversion is evaluated by maintaining the upper boundary temperature at 0°C. His results show that the onset of convection is dependent on the two thermal parameters which are functions of the boundary temperatures and the coefficients representing the density-temperature ranges considered ((0° to 20°C), (0° to 35°C) and (0° to 60°C)). He has considered three sets of the parameters r_1, r_2, and λ_1 and λ_2, are evaluated. The results show that the effect of density inversion on heat transfer rate is quite significant and to decrease as the temperature difference across the layer increases. For small ΔT, the heat transfer is found to be sufficiently small when compared to the non-density inversion situation.

Ramilison and Gebhart (1980) have studied buoyancy induced transport in porous media saturated with pure or saline water at low temperatures by using an accurate and much simpler density equation which applies to both pure and saline water to a pressure level of 1000 bars, at 20°C. They have considered vertical buoyancy driven plane flows imbedded in an extensive porous medium saturated with either pure or saline water under conditions in which density extremum might occur. The authors have neglected salinity diffusion, Dufour and Soret effects for small wall-to-ambient temperature differences. The necessary and sufficient conditions for the existence of similarity solutions are determined. They have investigated the region where buoyancy force reversal occurs, and the values of R correspond to small flow reversals. The authors found the instability of the numerical routine which they adopted in the range 0.195$< R <$0.4, in a region of large reversals. The authors found a large decrease in heat transfer as the buoyancy force reversal region is approached from each side. They observed the effects of convective inversion in the form of the temperature distributions to be relatively small.

Encouraged by the earlier two studies (Bejan 1980a, b) of penetrative convection in porous media, viz., (i) the vertical penetration into a well filled with porous medium, with application to a grain storage problem and (ii) the lateral penetration into a horizontal porous structure, with application to the natural convection cooling of rotating electric windings, Kenneth Blake, Andrian Bejan and Poulikakos (1984) have made a numerical study of two-dimensional natural convection in a horizontal porous layer heated from below and saturated with cold water with the objective to document numerically the characteristics of natural circulation in a porous layer heated from below and also to illustrate the high number characteristics of the phenomenon. They have considered a simple model of the moist ground trapped water under a layer of ice in winter. The layer is bounded above and below by solid walls maintained at temperature T_c and T_H such that $T_H > T_C$. The boundary temperatures are such that they embrace the density maximum of pure water at atmospheric pressure. The vertical boundaries located at $x = 0$ and $x = L$ are assumed to be impermeable and adiabatic. The authors have considered parabolic density temperature relationship, and three separate series of numerical simulations document the effect of Rayleigh number, bottom surface temperature and the horizontal length of the porous layer on the overall heat transfer rate vertically through the layer. In their study the governing equations were discretized by using the control volume formulation as given by Patankar (1980). The region of interest was covered with an array (m-2 by n-2) of square control volumes. The four boundaries were covered with control volumes of zero thickness. They applied the power-law scheme to determine both the heat and mass fluxes across each of the control volume boundaries. Their investigation predicts that the flow is multicellular, and the actual number of cells depends on the Rayleigh number. The effect of cell multiplication on the Nusselt number was also determined. They observed that as the bottom temperature T_H approaches 3.98°C, i.e., as the potentially unstable region vanishes, the natural circulation disappears. They also examined the effect of geometric aspect ratio on the flow and temperature patterns and found that the lateral extent of the porous layer as a relatively weak impact on the local character of the flow.

Recent reviews of natural convection in porous media demonstrate that the enclosed flows are becoming a classical subfield. Poulikakos and Andrian Bejan (1984) studied the penetrative convection in porous medium bounded by a horizontal wall with hot and cold spots numerically. Their study focuses on a semi-infinite isothermal porous medium heated and cooled from below periodically. Their study reports a series of numerical simulations and a scale analysis of the penetrative convection occurring along the unevenly heated horizontal wall of a semi-infinite porous medium. The porous medium is assumed to be locally in thermal equilibrium with the solid porous matrix. The authors have considered two simple functions for plate temperature viz., cosine variation and the step function.

Their investigation records the following observations

i) When the horizontal wall temperature varies between alternating hot and cold spots, the natural circulation consists of a row of counter-rotating cells situated near the wall. Each cell penetrates vertically into the porous medium to a distance approximately equal to $\lambda\sqrt{R_{ah}}$ where λ is the distance between a hot spot and the adjacent cold spot and R_{ah} is the Darcy-modified Rayleigh number.

ii) The ability of each cell to convert heat between two adjacent spots increases with the Rayleigh number.

iii) Each cell is a deformed plume in the sense that the hot plume rising above a hot spot be eventually turned around and sucked into the vacuum created around the closest cold spot.

iv) The heat transfer rate between two adjacent spots increases monotonically as the Rayleigh number R_{ah} increases

The configuration considered by the authors is most relevant in understanding the behaviour of underground layers heated unevenly.

Sudhakar (1993) has made a detailed study of penetrative convection in a porous layer in the presence of magnetic field. The boundaries are considered to be stress-free and perfect conductors of heat and the layer is a densely packed porous layer. He has employed the modified power integral technique, and his results predict that the critical penetration Rayleigh number increases more rapidly with the porous parameter than the Chandrasekhar number. The author has shown that for very large values of the porous parameter, there exits double are multi cells depending on the proper choice of the parameters. Another interesting result of his investigation is that Penetration Rayleigh number based on the thickness of the unstable layer attains a local minimum at λ=1.6. However, the penetration Rayleigh number based on the total depth of the fluid layer continuously increases with λ.

The author has made a detailed investigation of penetrative convection in sparsely and densely packed porous layers. He has employed Fourier-Transform technique and also Runge-Kutta Gill method for the purpose of determining the solution. He has predicted the conditions for the existence of rolls and squares and the results are discussed.

Srimani and Sudhakar (1996) have made a very detailed study of linear and nonlinear penetrative convection in a porous layer by considering a non-linear density temperature relationship.

The porous layer is considered to be densely packed. They have made a detailed analysis of both linear and nonlinear theories. They have presented the heat transport curve for a wide range of the porous parameter. They have used the modified power integral technique. Their results record the following aspects of the problem:

i) The relaxation of the upper boundary condition results in a thick stable layer.

ii) The increase in the available potential energy increases the upper boundary temperature to 8°C and further deepening of the stable layer is reduced.

iii) Multicell will be formed only when there is the transfer of kinetic energy from the unstable layer and this suggest that the boundary temperature should be less than 8°C.

iv) The motion is supercritical as in the case of the ordinary porous convection and hence 2D-rolls are preferred.

Review - Type III

The multi-component onset of convection is important in many naturally occurring phenomena and technological processes. In double diffusive convection, heat and solute diffuse at different rates, as a result of which complex flow structures may form which have no counterpart in buoyant flows driven by a single component. Extensive literature pertaining to this phenomenon is available (Fujita and Gosting (1956); Stern (1960); Miller (1966); Miller et al. (1967); Nield (1968); Hurle and Jakeman (197 1); Huppert and Manins (1972); Schechter et al (1972); Velarde and Schechter (1972); Vitaliano et al. (1972); Caldwell (1974); Turner (1974,1985); Griffiths (1979); Antoranz and Venard (1979); Leaist and Lyons (1980); Placsek and Toomre (1980); Narusawa and Suzukawa (1981); Srimani (1981); Takao, Tsuchiya and Narasuwa (1982); McTaggart (1983); Srimani (1984, 1991); Torrones and Pearistein (1989); Anamika (1990); Chen and Su (1992); Zimmermann Muller and Davis (1992); Tanny, Chen and Chen (1994); Shivakumar (1997); Skarda, Jacqmin and McCaughan (1998);).But in Type Ill, considered in this review, an additional effect viz., the effect of the coupled fluxes of the two properties due to irreversible thermodynamic effects is considered. This is termed as the effect of Coupled diffusion or Cross-diffusion and Sort effect is an example of this cross- diffusion where a flux of salt is caused by a spatial gradient of temperature. In fact, Dufour effect is the corresponding flux of heat caused by a spatial gradient of temperature.

Some literature (Hurle and Jakeman (1971); Skarda, Jacqmin and McCaughan (1998); Zimmermann et al (1992); Chen and Chen (1994)) in this direction are useful.McDougall (1983) has made a detailed study of Double-diffusive convection caused by coupled molecular diffusion. The results of his linear stability analysis, predicts that for a sufficiently large, coupled diffusion effect, fingers can form even when the concentrations of both the components make the fluid's density gradient statically stable. The conditions for the occurrence of finger as well diffusive instabilities are predicted. The results of his finite amplitude analysis reveal that for sufficiently large cross-diffusion effect, fingers do exist.

Absolutely very sparse literature is available in this direction and no literature is available for a heterogenous fluid layer.

The works discussed so far deal with penetrative convection in fluid and porous layers in absence of fixed-flux conditions. Tothe author's knowledge no literature pertaining to penetrative convection subject to nonlinear temperature/salinity profile with nonlinear density temperature and/are salinity relationship is available under different constraints in presence of fixed-flux conditions. Sparse literature with some common subject is available. Therefore, an attempt is made in this Review to include all these effects with the object of providing the prevailing influences of the relevant physical parameters on the stability of the system as well as on the bifurcation and fluid patterns.

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