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EXPRESSION OF THE FACT THAT PERTURBATIONS OF THE CAUCHY DIRICHLET PROBLEM ARE PROPAGATED WITH INFINITE SPEED

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ABSTRACT:
The paper deals with the Cauchy-Dirichlet problem for the nonlinear inhomogeneous diffusion equation with the possible power degeneration conditions near the boundary of a cone-like domain. Our main technical tool for the obtaining of solution estimations is a suitable weighted Nerenberg-Gagliardo type inequality, which in turn is connected to a weighted isoperimetric inequality characterizing the geometry of the domain. On this basis we study the property of infinite speed of the perturbation propagation of the solution. The sufficient conditions ensuring the possibility to estimate the radius of a solution support in the absence of a source are given. The existence of a strong generalized solution has been proved.
KEYWORDS:-cauchy-dirichlet, technical, perturbation, infinite, speed

INTRODUCTION:

The One-Dimensional Wave Equation • Equation (1) utt − c2(x, t)uxx = f(x, t) is called the one-dimensional wave equation. – The coefficient c has the dimension of a speed and in fact,[1,2] we will shortly see that it represents the wave propagation along the string. – When f ≡ 0, the equation is homogeneous and the superposition principle holds: if u1 and u2 are solutions of (2) utt − c2uxx = 0 and a, b ∈ R, then au1 + bu2 is also a solution. – More generally, if uk(x, t) is a family of solutions depending on the parameter k and g = g(k) is a function rapidly vanishing at infinity, then X∞ k=1 uk(x, t)g(k) and Z +∞ −∞ uk(x, t)g(k)dk are still solutions of (2). • Suppose we are considering the space-time region 0 < x < L, 0 < t < T. By analogy with the Cauchy problem for second order o.d.e., the second order derivative in (1) suggests that in a well-posed problem for the (one-dimensional) wave equation not only the initial profiles of the string but the initial velocity has to be assigned as well. – Thus, our initial (or Cauchy) data are u(x, 0) = g(x), u)=h(x), x ∈ [0, L]. • The boundary data are typically: (i) Dirichlet data describes the displacement of the end points of the string: u(0, t) = a(t), u(L, t) = b(t), t> 0. If a(t) = b(t) ≡ 0 (homogeneous data), both ends are fixed, with zero displacement.[3,4] (ii) Neumann data describes the applied (scalar) vertical tension at end points. AS in the derivation of the wave equation, we may model this tension by τ0ux, so that the Neumann conditions takes the form τ0ux(0, t) = a(t), τ0ux(L, t) = b(t), t> 0. In the special case of homogeneous data, a(t) = b(t) ≡ 0, both ends of the string are attached to a frictionless sleeve and are free to move vertically. (iii) Robin data describes a linear elastic attachment at end points. One way to realize this type of boundary condition is to attach an end point to a linear spring whose other end is fixed. - This translates into assigning τ0ux(0, t) = ku(0, t), τ0(L, t) = −ku(L, t), t> 0, where k > 0 is the elastic constant of the spring.[5,6]

1. The Global Cauchy problem for An Infinite String. We may think of a string of infinite length and assign only initial data u(x, 0) = g(x), ut(x, 0) = h(x), x ∈ R. Although physically unrealistic, it turns out that the solution of the global Cauchy problem is of fundamental importance. The solution of the global Cauchy problem is given by the d'Alembert formula u(x, t) = 1 2 [g(x − ct) + g(x + ct)] + Z x+ct x−ct h(y)dy. 2. The Semifinite String. The initial-boundary problem is utt − c2uxx = 0, x> 0, t> 0 u(x, 0) = g(x), ut(x, 0) = h(x), x ≥ 0 u(0, t) =

0 (fixed endpoint), or ux(0, t) = 0 (free endpoint), t ≥ 0, where c2 = τ0/ρ0 is constant. • The problem for the semi-infinite string can be reduced to the problem for the infinite string, so that the solution of the problem for the infinite string, when restricted to a half-line, yields the solution of the problem for the semiinfinite string. • To do this, we must extend the initial condition to the entire line in such a way that the solution satisfies the boundary conditions at the point x = 0. • Parity considerations are helpful in doing this. Case I. The initial condition can be extended to the entire line as an even function provided the relation ux x=0 = 0 holds. But will the solution be an even function of x at all times? – The wave equation is invariant under the transformation x 7→ −x.[7,8] If the initial condition is even, that is, also invariant under this transformation, we then have two solutions u(x, t) and u(−x, t)of a global Cauchy problem. – However, the uniqueness of the solution of the global Cauchy problem was proved in the derivation of d'Alembertformula[9,10]

DISCUSSION:

Hence the two solutions u(x, t) and u(−x, t) coincide, so that u(x, t) = u(−x, t), and the solution is an even function. • This is a general idea: If the problem possesses some symmetry and the solution is unique, then the solution must also possess the symmetry. Case II. We can use an odd extention when the condition u x=0 = 0 holds. The Cauchy-Dirichlet Problem for The Finite String. Suppose that the vibration of a violin chord is modelled by the following CauchyDirichlet problem utt − c2uxx = 0, 0 < x < L, t > 0 u(x, 0) = g(x), ut(x, 0) = h(x), 0 ≤ x ≤ L, u(0, t) = u(L, t)=0, t ≥ 0, where c2 = τ0/ρ0 is constant. • The d'Alembert's method is not very convenient for solving the boundary value problem in the case of finite string. Later on we will develop another very powerful method to handle this case.[9,10] • At present we illustrate the applications of d'Alembert's method[11,12] to the problem utt − c2uxx = 0, 0 < x < L, t > 0 u(x, 0) = g(x), ut(x, 0) = h(x), 0 ≤ x ≤ L, u(0, t) = k(t), u(L, t)=0, t ≥ 0, The values brought from the boundary and the initial interval t = 0 along the charcteristics x − ±ct =constant contribute to the solution at the point (x, t). The characteristics undergo bending as they reflected from the boundary. – As a result, the value of the solution is an alternating sum of the values at the nodes of the resulting broken lines. Uniqueness. Use conservation of energy. – Let u and v be solutions of utt − c2uxx = f(x, t), 0 < x < L, t > 0 u(x, 0) = g(x), ut(x, 0) = h(x), 0 ≤ x ≤ L, u(0, t) = k1(t), u(L, t) = k2(t), t ≥ 0, Then w = u – v
is a solution of the problem utt − c2uxx = 0, 0 < x < L, t > 0 u(x, 0) = 0, ut(x, 0) = 0, 0 ≤ x ≤ L, u(0, t)=0, u(L, t)=0, t ≥ 0. Claim: w ≡ 0. – The total mechanical energy E(t) = Ekin(t) + Epot(t) = 1 2 Z L 0 [ρ0w2 t + τ0w2 x]dx and in our case we have E˙(t)=0 since f = 0 and wt(x, 0) = wx(x, 0) = 0, whence E(t) = E(0), ∀t ≥ 0. Since, on particular, wt(x, 0) = wx(x, 0) = 0, we have E(t) = E(0) = 0, ∀t ≥ 0. On the other hand, Ekin(t) ≥ 0, Epot(t) ≥ 0, so that we deduce Ekin(t)=0, Epot(t)=0, which forces wt = wx = 0. Therefore w is constant. – Since w(x, 0) = 0, we conclude that w(x, t)=0, ∀t ≥ 0. Remark. (i) If we pluck a violin chord at a point, the initial profile is continuous but has a corner at that point and cannot be even C1. The physically realistic assumption for the initial profile g is continuity. (ii) Similarly, if we model the vibration of a chord set into motion by a strike or a little hammer, we should allow discontinuity in the initial velocity. • Observe that d'Alembert formula makes perfect sense even for g continuous and h bounded. – The question is in which sense the resulting function satisfies the wave equation, since, in principle, it is not even differentiable, only continuous. • It is possible to introduce weak formulations of the various initial-boundary value problem, in order to include realistic initial data and solutions with a low degree of regularity.[13]

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RESULTS:

In cylindrical polar coordinates (R, θ, z), the wave equation (3.1.1) assumes the form uRR + 1 R uR + 1 R2 uθθ + uzz = 1 c 2 utt
uRR + 1 R uR = 1 c 2 utt.

In general, it is not easy to find the solution of (5.11.1). However, we shall solve this equation by using the method of separation of variables in Chapter 7. Here we derive the solution for outgoing cylindrical waves from the spherical wave solution (5.10.8). We assume that sources of constant strength Q (t) per unit length are distributed uniformly on the z-axis. The solution for the cylindrical waves produced by the line source is given by the total disturbance u (R, t) = − 1 4π ∞ −∞ 1 r Q t − r c dz = − 1 2π ∞ 0 1 r Q t − r c dz, (5.11.3) where R is the distance from the z-axis so that R2 = r 2 − z 2 . Substitution of z = R sinh ξ and r = R cosh ξ in (5.11.3) gives u (R, t) = − 1 2π ∞ 0 Q t − R c cosh ξ dξ. (5.11.4) This is usually considered as the cylindrical wave function due to a source of strength Q (t) at R = 0. It follows from (5.11.4) that utt = − 1 2π ∞ 0 Q ′′

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t − R c cosh ξ dξ, (5.11.5) uR = 1 2πc ∞ 0 cosh ξ Q′ t − R c cosh ξ dξ, (5.11.6) uRR = − 1 2πc2 ∞ 0 cosh2 ξ Q′′

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t − R c cosh ξ dξ, (5.11.7) which give c 2 uRR + 1 R uR − utt = 1 2π ∞ 0 d dξ c R Q ′ t − R c cosh ξ sinh ξ dξ = lim ξ→∞ c 2πR Q ′ t − R c cosh ξ sinh ξ = 0, provided the differentiation under the sign of integration is justified and the above limit is zero. This means that u (R, t) satisfies the cylindrical wave equation[11,12]

"Since a general solution must be judged impossible from want of analysis, we must be content with the knowledge of some special cases, and that all the more, since the development of various cases seems to be the only way to bringing us at last to a more perfect knowledge." Leonhard Euler "What would geometry be without Gauss, mathematical logic without Boole, algebra without Hamilton, analysis without Cauchy?" George Temple

In the theory of ordinary differential equations, by the initial-value problem we mean the problem of finding the solutions of a given differential equation with the appropriate number of initial conditions prescribed at an initial point. For example, the second-order ordinary differential equation d 2u dt2 = f t, u, du dt and the initial conditions u (t0) = α,

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du dt (t0) = β, constitute an initial-value problem. An analogous problem can be defined in the case of partial differential equations. Here we shall state the problem involving second-order partial differential equations in two independent variables.

The preceding statement seems equally applicable to hyperbolic, parabolic, or elliptic equations. However, we shall see that difficulties arise in formulating the Cauchy problem for nonhyperbolic equations. Consider, for instance, the famous Hadamard (1952) example. The problem consists of the elliptic (or Laplace) equation uxx + uyy = 0, and the initial conditions on y = 0 u (x, 0) = 0, uy (x, 0) = n −1 sin nx. The solution of this problem is u (x, y) = n −2 sinhny sin nx, which can be easily verified. It can be seen that, when n tends to infinity, the function n −1 sin nx tends uniformly to zero. But the solution n −2 sinhny sin nx does not become small, as n increases for any nonzero y. Physically, the solution represents an oscillation with unbounded amplitude n −2 sinhny as y → ∞ for any fixed x. Even if n is a fixed number, this solution is unstable in the sense that u → ∞ as y → ∞ for any fixed x for

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which sin nx = 0. It is obvious then that the solution does not depend continuously on the data. Thus, it is not a properly posed problem. In addition to existence and uniqueness, the question of continuous dependence of the solution on the initial data arises in connection with the Cauchy–Kowalewskaya theorem. It is well known that any continuous function can accurately be approximated by polynomials. We can apply the Cauchy–Kowalewskaya theorem with continuous data by using polynomial approximations only if a small variation in the initial data leads to a small change in the solution.

CONCLUSIONS:

dx2 − c 2 dt2 = 0, which reduces to dx + c dt = 0, dx − c dt = 0. The integrals are the straight lines x + ct = c1, x − ct = c2. Introducing the characteristic coordinates ξ = x + ct, η = x − ct, we obtain uxx = uξξ + 2 uξη + uηη, utt = c 2 (uξξ − 2 uξη + uηη). Substitution of these in equation (5.3.1) yields −4c 2uξη = 0. Since c = 0, we have uξη = 0. Integrating with respect to ξ, we obtain uη = ψ ∗ (η), where ψ ∗ (η) is an arbitrary function of η. Integrating again with respect to η, we obtain u (ξ,η) = ψ ∗ (η) dη + φ (ξ). If we set ψ (η) = * ψ ∗ (η) dη, we have u (ξ,η) = φ (ξ) + ψ (η), where φ and ψ are arbitrary functions. Transforming to the original variables x and t, we find the general solution of the wave equation
φ (x) = 1 2 f (x) + 1 2c x x0 g (τ) dτ + K 2 , ψ (x) = 1 2 f (x) − 1 2c x x0 g (τ) dτ − K 2 . The solution is thus given by u (x, t) = 1 2 [f (x + ct) + f (x − ct)] + 1 2c x+ct x0 g (τ) dτ − x−ct x0 g (τ) dτ = 1 2 [f (x + ct) + f (x − ct)] + 1 2c x+ct x−ct g (τ) dτ.

By direct substitution, it can also be shown that the solution (5.3.8) is uniquely determined by the initial conditions (5.3.2) and (5.3.3). It is important to note that the solution u (x, t) depends only on the initial values of f at points x − ct and x + ct and values of g between these two points. In other words, the solution does not depend at all on initial values outside this interval, x − ct ≤ x ≤ x + ct. This interval is called the domain of dependence of the variables (x, t). Moreover, the solution depends continuously on the initial data, that is, the problem is well posed. In other words, a small change in either f or g results in a correspondingly small change in the solution u (x, t). Mathematically, this can be stated as follows: For every ε > 0 and for each time interval 0 ≤ t ≤ t0, there exists a number δ (ε, t0) such that u (x, t) − u ∗ (x, t)| < ε, whenever |f (x) − f ∗ (x)| < δ, |g (x) − g ∗ (x)| < δ. The proof follows immediately from equation (5.3.8). We have |u (x, t) − u ∗ (x, t)| ≤ 1 2 |f (x + ct) − f ∗ (x + ct)| + 1 2 |f (x − ct) − f ∗ (x − ct)| + 1 2c x+ct x−ct |g (τ) − g ∗ (τ)| dτ< ε, where ε = δ (1 + t0). For any finite time interval 0 <t[13]

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