

# Mathematical Modeling and Stochastic Analysis of textile Wool Industry

Pardeep Kumar<sup>1</sup>, Mimansha<sup>2</sup>

<sup>1</sup>Assistant Professor, Department of Mathematics, Chhotu Ram Arya College Sonipat,  
Haryana

<sup>2</sup>Assistant Professor, Department of Mathematics, Tika Ram P. G. College Sonipat, Haryana

<sup>1</sup>[deepsaroha1@gmail.com](mailto:deepsaroha1@gmail.com); <sup>2</sup>[mims.sehrawat2094@gmail.com](mailto:mims.sehrawat2094@gmail.com)

**DOI:** 10.6084/m9.figshare.19881577

**Corresponding Author: Mimansha**

**ABSTRACT:**The steady-state stochastic analysis of the wool industry is studied using RPGT. There are four subsystems in the textile wool industry. Carding (A), Spinning (B), Weaving (C), and Finishing (D) are the four steps in the carding process (D). The failure state of subsystems is determined via fuzzy logic. All subsystems are served by a single server (repairman). Subsystems A and B receive priority in repair. System behavior is discussed using exponential failure rates and general repair rates. Statistics show that failure and repair are unrelated. The Regenerative Point Graphical Technique is used to evaluate several metrics such as mean time to system failure, system availability, server busy period, and predicted number of server visits. Profit optimization is also discussed. Graphs and tables are used to illustrate behavior of a system.

**Keywords:** Profit optimization, MTSF, Availability

## 1. INTRODUCTION:

The behavioral analysis of a textile wool industry in Punjab for steady-state utilizing RPGT is presented in this research. Poonam (2018) studied sensitivity analysis of a biscuit mill using RPGT, taking into account the value of individual components in a system. The cold standby strategy with priority for PM was calculated by Kumar et al. (2019). Kumar et al. (2017) used RPGT to compute the edible oil refinery industry. Kumar et al. (2018) used RPGT under steady-state to determine the behavior of a bread-making organization with five separate subsystems such as the oven, tunnels, mixer, divider, and proofer.

Anchal et al. (2021) discussed the SRGM classical employing difference equation, in which binary categories of faults: simple and hard with respect to the period in which they develop for isolation and elimination after their detection has been obtained. Kumar et al. (2019) presented an RPGT behavioral analysis of a paper mill washing unit. Kumari et al. (2021) used RPGT to analyses the profit of an agricultural thresher facility. The 3:4: G Organization was proposed by Kumar et al. (2018). Carding (A), Spinning (B), Weaving (C), and Finishing (D) are the four subsystems of the textile wool business (D). Carding (A) and Spinning Manufacturing (B) have parallel units, whereas Weaving (C) and Finishing (D) have series units. As a result, subsystems A and B can fail totally in a reduced condition, while subsystem C can fail directly. The failure state of subsystems is determined via fuzzy logic. All subsystems are served by a single server (repairman). If the subsystem 'C' fails, the knitting is stopped and only textile wool is produced for marketing purposes, resulting in a smaller system.

A transition diagram of the system is created using various probabilities to determine Primary Circuits, Secondary Circuits, Tertiary Circuits, and Base-State. RPGT is used to analyses several factors such as mean time to system failure, system availability, server busy period, and expected number of server visits. Profit optimization is also discussed. Graphs and tables are used to illustrate system behavior.

## 2. ASSUMPTIONS& Notations:

- Single repair facility is available 24\*7.
- Failures and repairs are statistically independent.
- When any subsystem fails then the system is in failed state.

A/B/C : Units in full capacity working

$\lambda_i$  : Constant failure rate of the units

$w_i$  : Constant repair rate of the units

## 3. TRANSITION DIAGRAM OF THE SYSTEM :

---

Following the above assumption and notations the Transition Diagram of the system is given in Figure 1.

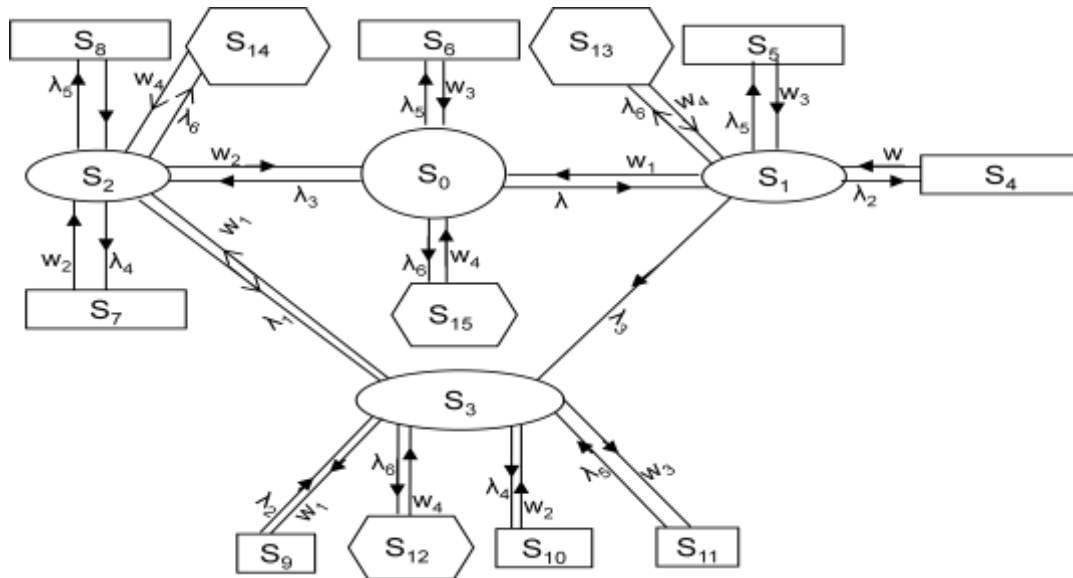


Fig. 1: Transition Diagram

#### 4. Transition Probability & Mean Sojourn Times

Table 1: Transition Probability

$q_{ij}^{(t)}$	$P_{ij} = q_{ij}^{*(t)}$
$q_{0,1} = \lambda_1 e^{-(\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{0,1} = \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{0,2} = \lambda_3 e^{-(\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{0,2} = \lambda_3 / (\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{0,6} = \lambda_5 e^{-(\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{0,6} = \lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{0,15} = \lambda_6 e^{-(\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{0,15} = \lambda_6 / (\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{1,0} = w_1 e^{-(w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{1,0} = w_1 / (w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{1,3} = \lambda_3 e^{-(w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{1,3} = \lambda_3 / (w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{1,4} = \lambda_2 e^{-(w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{1,4} = \lambda_2 / (w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{1,5} = \lambda_5 e^{-(w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{1,5} = \lambda_5 / (w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{1,13} = \lambda_6 e^{-(w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)t}$	$p_{1,13} = \lambda_6 / (w_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6)$
$q_{2,0} = w_2 e^{-(w_2 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6)t}$	$p_{2,0} = w_2 / (w_2 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6)$
$q_{2,3} = \lambda_1 e^{-(w_2 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6)t}$	$p_{2,3} = \lambda_1 / (w_2 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6)$

$q_{2,7} = \lambda_4 e^{-(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{2,7} = \lambda_4/(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)$
$q_{2,8} = \lambda_5 e^{-(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{2,8} = \lambda_5/(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)$
$q_{2,14} = \lambda_6 e^{-(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{2,14} = \lambda_6/(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)$
$q_{3,2} = w_1 e^{-(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{3,2} = w_1/(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)$
$q_{3,9} = \lambda_2 e^{-(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{3,9} = \lambda_2/(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)$
$q_{3,10} = \lambda_4 e^{-(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{3,10} = \lambda_4/(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)$
$q_{3,11} = \lambda_5 e^{-(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{3,11} = \lambda_5/(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)$
$q_{3,12} = \lambda_6 e^{-(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)t}$	$p_{3,12} = \lambda_6/(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)$
$q_{4,1} = w_1 e^{-w_1 t}$	$p_{4,1} = w_1/w_1 = 1$
$q_{5,1} = w_3 e^{-w_3 t}$	$p_{5,1} = w_3/w_3 = 1$
$q_{6,0} = w_3 e^{-w_3 t}$	$p_{6,0} = 1$
$q_{7,2} = w_2 e^{-w_2 t}$	$p_{7,2} = 1$

**Table 2: Mean Sojourn Times**

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_0^{(t)} = e^{-(\lambda_1+\lambda_3+\lambda_5+\lambda_6)t}$	$\mu_0 = 1/(\lambda_1+\lambda_3+\lambda_5+\lambda_6)$
$R_1^{(t)} = e^{-(w_1+\lambda_2+\lambda_3+\lambda_5+\lambda_6)t}$	$\mu_1 = 1/(w_1+\lambda_2+\lambda_3+\lambda_5+\lambda_6)$
$R_2^{(t)} = e^{-(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)t}$	$\mu_2 = 1/(w_2+\lambda_1+\lambda_4+\lambda_5+\lambda_6)$
$R_3^{(t)} = e^{-(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)t}$	$\mu_3 = 1/(w_1+\lambda_2+\lambda_4+\lambda_5+\lambda_6)$
$R_4^{(t)} = e^{-w_1 t}$	$\mu_4 = 1/w_1$
$R_5^{(t)} = e^{-w_3 t}$	$\mu_5 = 1/w_3$
$R_6^{(t)} = e^{-w_3 t}$	$\mu_6 = 1/w_3$
$R_7^{(t)} = e^{-w_2 t}$	$\mu_7 = 1/w_2$
$R_8^{(t)} = e^{-w_3 t}$	$\mu_8 = 1/w_3$
$R_9^{(t)} = e^{-w_1 t}$	$\mu_9 = 1/w_1$
$R_{10}^{(t)} = e^{-w_2 t}$	$\mu_{10} = 1/w_2$

$R_{11}^{(t)} = e^{-w_3 t}$	$\mu_{11} = 1/w_3$
$R_{12}^{(t)} = e^{-w_4 t}$	$\mu_{12} = 1/w_4$
$R_{13}^{(t)} = e^{-w_4 t}$	$\mu_{13} = 1/w_4$
$R_{14}^{(t)} = e^{-w_4 t}$	$\mu_{14} = 1/w_4$
$R_{15}^{(t)} = e^{-w_4 t}$	$\mu_{15} = 1/w_4$

### 5. Transition Probabilities

The MTSF and all the key parameters of the organization under steady state conditions are evaluated, applying RPGT and by ‘3’ as the base-state of the scheme as under:

$$V_{3,0} = (3,2,0)/\{1-(2,7,2)\}\{1-(2,8,2)\}\{1-(2,14,2)\}[1-(2,0,2)/\{1-(0,6,0)\}\{1-(0,15,0)\}\{1-(0,1,0)\}\{1-(0,6,0)\}\{1-(0,15,0)\}[1-(0,1,0)/\{1-(1,4,1)\}\{1-(1,5,1)\}\{1-(1,13,1)\}]\}$$

$$V_{3,1} = (3,2,0,1)/\{1-(2,7,2)\}\{1-(2,8,2)\}\{1-(2,14,2)\}[1-(2,0,2)/\{1-(0,6,0)\}\{1-(0,15,0)\}\{1-(0,1,0)\}\{1-(0,6,0)\}\{1-(0,15,0)\}[1-(0,1,0)/\{1-(1,4,1)\}\{1-(1,5,1)\}\{1-(1,13,1)\}]\}\{1-(1,4,1)\}\{1-(1,5,1)\}\{1-(1,13,1)\}$$

$$V_{3,2} = \dots \text{Continuous}$$

**MTSF (T<sub>0</sub>):** Regenerative un-failed states to which the classification can transit(initial state ‘0’), formerly inflowing any unsuccessful state are: ‘i’ = 0,1,2,3,12,13,14,15.

$$T_0 = [\mu_0 + p_{0,15}\mu_{15} + \mu_1 p_{0,1}/(1-p_{13,1}p_{1,13}) + \mu_2 \{ p_{0,2}(1-p_{3,12}p_{12,3})/(1-p_{3,12}p_{12,3}-p_{2,3}p_{3,2}) + p_{0,1}p_{1,3}p_{3,2}/(1-p_{13,1}p_{1,13})(1-p_{3,12}p_{12,3})(1-p_{2,14}p_{14,2}-p_{3,2}p_{2,3}) \} + \mu_3 \{ p_{0,1}p_{1,3}(1-p_{2,14}p_{2,14})/(1-p_{1,13}p_{13,1})(1-p_{3,12}p_{12,3})(1-p_{2,14}p_{14,2}-p_{3,2}p_{2,3}) + p_{0,2}p_{2,3}/(1-p_{2,14}p_{14,2})(1-p_{2,3}p_{3,2}-p_{3,12}p_{12,3}) \} + \{ p_{0,1}p_{1,3}p_{3,12}(1-p_{2,14}p_{14,2})/(1-p_{1,13}p_{13,1})(1-p_{3,12}p_{12,3})(1-p_{2,14}p_{14,2}-p_{3,2}p_{2,3}) + p_{0,2}p_{2,3}p_{3,12}/(1-p_{2,14}p_{14,2})(1-p_{3,12}p_{12,3}-p_{2,3}p_{3,2}) \} \mu_{12} + \{ p_{0,1}p_{1,13}/(1-p_{13,1}p_{1,13}) \} \mu_{13} + \{ p_{0,2}p_{2,14}(1-p_{3,12}p_{12,3})/(1-p_{2,14}p_{14,2})(1-p_{2,3}p_{3,2}-p_{3,12}p_{12,3}) + p_{0,1}p_{1,3}p_{3,2}p_{2,14}/(1-p_{1,13}p_{13,1})(1-p_{3,12}p_{12,3})(1-p_{2,14}p_{14,2}-p_{3,2}p_{2,3}) \} \mu_{14}]/[1-p_{0,1}p_{1,0}/(1-p_{13,1}p_{1,13})-p_{0,15}p_{15,0}-p_{0,2}p_{2,0}(1-p_{3,12}p_{12,3})/(1-p_{2,14}p_{14,2})-p_{0,1}p_{1,3}p_{3,2}p_{2,0}/(1-p_{1,13}p_{13,1})(1-p_{2,14}p_{14,2}-p_{3,2}p_{2,3})]$$

**Availability of the System (A<sub>0</sub>):** Regenerative states at which the organization is accessible are ‘j’ = 0, 1, 2,3,12,13,14,15 and ‘i’ = 0 to 15

$$A_0 = (V_{3,0}f_0\mu_0 + V_{3,1}f_1\mu_1 + V_{3,2}f_2\mu_2 + V_{3,3}f_3\mu_3 + V_{3,12}f_{12}\mu_{12} + V_{3,13}f_{13}\mu_{13} + V_{3,14}f_{14}\mu_{14} + V_{3,15}f_{15}\mu_{15})/(V_{3,0}\mu_0^1 + V_{3,1}\mu_1^1 + V_{3,2}\mu_2^1 + V_{3,3}\mu_3^1 + V_{3,4}\mu_4^1 + V_{3,5}\mu_5^1 + V_{3,6}\mu_6^1 + V_{3,7}\mu_7^1)$$

$$+V_{3,8}\mu_8^1+V_{3,9}\mu_9^1+V_{3,10}\mu_{10}^1+V_{3,11}\mu_{11}^1+V_{3,12}\mu_{12}^1+V_{3,13}\mu_{13}^1+V_{3,14}\mu_{14}^1+V_{3,15}\mu_{15}^1)$$

**Busy Period of Server (B<sub>0</sub>):** Regenerative states where the server is busy while doing repairs are ‘j’ = 1 to 15 and ‘i’ = 0 to 15.

$$B_0 = (V_{3,1}\eta_1+V_{3,2}\eta_2+V_{3,3}\eta_3+V_{3,4}\eta_4+V_{3,5}\eta_5+V_{3,6}\eta_6+V_{3,7}\eta_7+V_{3,8}\eta_8+V_{3,9}\eta_9+V_{3,10}\eta_{10}+V_{3,11}\eta_{11}+V_{3,12}\eta_{12}+V_{3,13}\eta_{13}+V_{3,14}\eta_{14}+V_{3,15}\eta_{15})/(V_{3,0}\mu_0^1+V_{3,1}\mu_1^1+V_{3,2}\mu_2^1+V_{3,3}\mu_3^1+V_{3,4}\mu_4^1+V_{3,5}\mu_5^1+V_{3,6}\mu_6^1+V_{3,7}\mu_7^1+V_{3,8}\mu_8^1+V_{3,9}\mu_9^1+V_{3,10}\mu_{10}^1+V_{3,11}\mu_{11}^1+V_{3,12}\mu_{12}^1+V_{3,13}\mu_{13}^1+V_{3,14}\mu_{14}^1+V_{3,15}\mu_{15}^1)$$

**Expected Number of Server’s Visits (V<sub>0</sub>):** The regenerative states where the server visits a fresh for repair of system are ‘j’ = 1,2,6,15 and ‘i’ = 0 to 15 for  $\xi = 3$ ,

$$V_0 = (V_{3,1}+V_{3,2}+V_{3,6}+V_{3,15})/(V_{3,0}\mu_0^1+V_{3,1}\mu_1^1+V_{3,2}\mu_2^1+V_{3,3}\mu_3^1+V_{3,4}\mu_4^1+V_{3,5}\mu_5^1+V_{3,6}\mu_6^1+V_{3,7}\mu_7^1+V_{3,8}\mu_8^1+V_{3,9}\mu_9^1+V_{3,10}\mu_{10}^1+V_{3,11}\mu_{11}^1+V_{3,12}\mu_{12}^1+V_{3,13}\mu_{13}^1+V_{3,14}\mu_{14}^1+V_{3,15}\mu_{15}^1)$$

**6. Particular Cases: -** Where  $\lambda_i = \lambda$  and  $w_i = w$

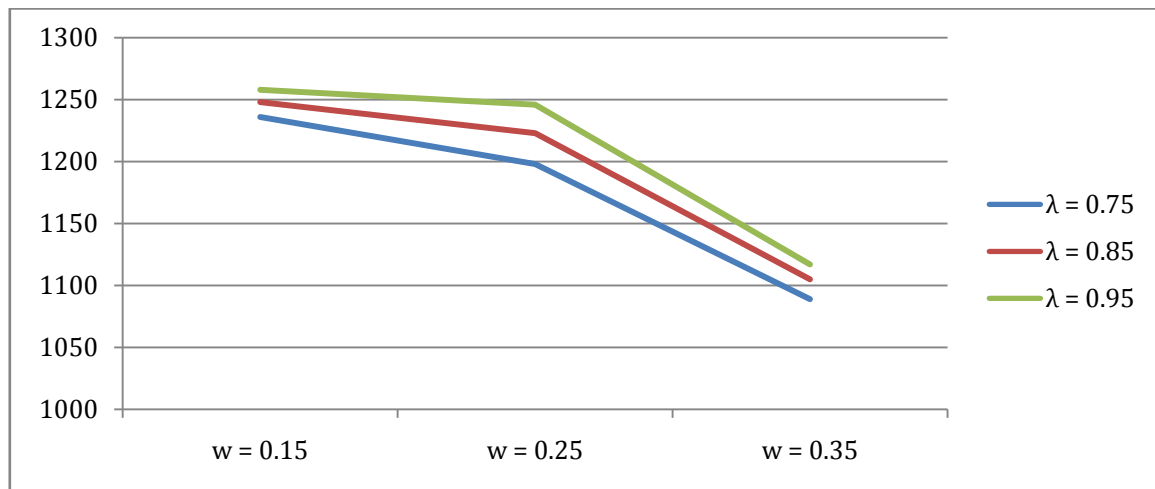
**Profit Function:** The system can be done by utilized profit function

$$P_0 = C_1A_0 - (C_2B_0 + C_3V_0) = C_1A_0 - C_2B_0 - C_3V_0$$

Where:  $C_1 = 1000$ ;  $C_2 = 50$ ;  $C_3 = 100$

**Table 3: Profit Function**

	$\lambda = 0.75$	$\lambda = 0.85$	$\lambda = 0.95$
w = 0.15	1236	1248	1258
w = 0.25	1198	1223	1246
w = 0.35	1089	1105	1117



**Fig. 2: Profit Function**

## 7. Conclusion

Fig. 2 and table 3, it is seen that profit increase with the expansion in repair rates for example cost is directly proportional to the repair rates of units, henceforth profit function is conversely proportional to the disappointment/failure rates.

## References: -

1. Kumari, S., Khurana, P., and Singla, S. (2021). Behaviour and profit analysis of a thresher plant under steady state. *International Journal of System Assurance Engineering and Management*, pp. 1-12.
2. Anchal, Majumder, A., and Goel, P. (2021). Irregular Fluctuation of Successive SW Release Models. *Design Engineering*, no. 7, pp. 8954-8962.
3. Kumar, A. Garg, D. and Goel, P. (2019). Mathematical modelling and behavioural analysis of a washing unit in a paper mill. *International Journal of System Assurance Engineering and Management*, 10, 1639-1645.
4. Poonam and Goel, P. (2018). Sensitivity analysis of a biscuit making plant. *International Journal of Statistics and Applied Mathematics*, 3(2), 598-606.
5. Kumar, A., Garg, D., and Goel, P. (2017). Mathematical modelling and profit analysis of an edible oil refinery industry. *Airo International Research Journal*, 12:1-8.

6. Kumar, A., Goel, P., and Garg, D. (2018). Behaviour analysis of a bread-making system. International Journal of Statistics and Applied Mathematics, vol. 3(6), pp. 56-61.
7. Kumar, A., Garg, D., and Goel, P. (2018). Sensitivity Analysis of 3:4: Good System. International Journal of Advance Research in Science and Engineering, 7(2):851-862.
8. Kumar, A., Garg, D., and Goel, P. (2019) Sensitivity analysis of a cold standby system with priority for preventive maintenance. Journal of Advances and Scholarly Researches in Allied Education 16, 253-258.