RESEARCHERID THOMSON REUTERS
International Journal of Research in Engineering and Applied Sciences(IJREAS)
Available online at http://euroasiapub.org/journals.php
Vol. 15 Issue 02, February -2025,
ISSN(O): 2249-3905, ISSN(P) : 2349-6525 | Impact Factor: 8.202| Thomson Reuters ID: L-5236-2015

AXIALLY SYMMETRIC BIANCHI TYPE – I STRING COSMOGICAL MODEL

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ABSTRACT:

The present paper provides string cosmological model in axially symmetric Bianchi type-I space time in absence and presence of magnetic field using different assumptions. Various physical and geometrical parameters have been also evaluated and discussed.

Key words: axial symmetry, string cosmology, space time, magnetic field, Shear scalar etc.

1. INTRODUCTION

In present days many researchers in theory of relativity have focussed their mind towards the study of string cosmologies [1-3, 7-10, 30]. Letelier [18] studied a gauge invariant model of a cloud formed by geometric strings and used this model as a source of the gravitational field. In particular, he solved the Einstein equations for a plane symmetric, a spherically symmetric, and particular case of cylindrically symmetric space-time [16, 17]. The presence of the strings in the early universe can be explained using grand unified theories [13, 32] (GUTS). Letelier [18] has studied model of a string cloud, in which the strings that form the cloud are massive strings instead of geometrical strings. Each massive string is formed by a geometric string with particles attached along its extension. Hence, the strings that form the cloud are the generalization of Takhbayaris's realistic model of strings which we call p-strings [15, 27].

As a matter of fact, at the early stage of the universe a phase transition occurs as the temperature lowers below some critical temperature and this can give rise to various topologically stable defects of which strings are of most important whose world sheets are two-dimensional time-like surfaces (Kibble [13]). It has been noted (Kibble [13]) that the existence of a large scalar network of strings in the early universe does not contradict the present-day observations of the universe and further the vacuum strings (Zeldovich [33]) can generate density fluctuations sufficient to explain the galaxy formation. These strings have stress energy

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and they couple to the gravitational field so that it may be interesting to study the gravitational effects which arise from strings. This has been already done by several authors, (Vilenkin [28], Gott [12], Garfinkle [11]), although the general relativistic treatment of strings was pioneered by Letelier [16] and Stachel [24].

In geometrical string (massless) models, infinite number of degrees of freedom are possessed by each string for which the end points move at the speed of light. This problem is resolved by considering the realistic (massive) string model of Takabayashi [27]. The energy-momentum tensor for the massive strings has been first formulated by Letelier [16], who considered the massive string being formed by geometric string with particles attached along its extension. Its application to general relativity first appeared in Letelier [18], while Stachel [24] considered massless strings. So, the total energy- momentum tensor for a cloud of massive strings can be written as

(1.1) $T_{\beta}^{\gamma} = \rho\ u_{\beta}u^{\gamma} - s\ x_{\beta}x^{\gamma}$

where ρ is the rest energy density for a cloud of strings with particles attached to them (p-strings). Thus, we have

(1.2) $\rho = \rho_p + s$

ρ_p being the particle energy density and s being the string's tension density, u^{β} is the four velocities for the cloud of particles and x^{β} is the four vector representing the string's direction which essentially is the direction of anisotropy. Thus

(1.3) $u_{\beta}u^{\beta} = -1 = -\ x_{\beta}x^{\beta}$ and $u_{\beta}x^{\beta} = 0$

in (-, +, +, +) singnature (i.e. +2 sign.)

Banerjee et. al. [5] have found some cosmological solutions in Bianchi I space time following the technique used by Letelier and Stachel with/without magnetic field. Melvin [19] in his solution for dust and deelectromagnetic field argued that the presence of magnetic field is not as unrealistic as it appears to be, because for a large part of the history of evolution matter was highly ionized, and matter and field were smoothly coupled. Later during cooling as a result of expansion the ions combined to form neutral matter. Some other workers in this line are Pradhan et.al [20], Reddy and Ramesh [22], Saha and Mihai [23], Kumar [14], Santhi et.al [25], Singh [26], and Yadav etal [31].

In this chapter we have studied the string cosmology in axially symmetric Bianchi I space time in absence and presence of magnetic field using different assumptions. Various physical parameters of the model have been also evaluated and discussed.

2. THE FIELD EQUATIONS: -

Here we take an axially symmetric Bianchi I model, written as

(2.1) $ds^2 = \exp(2\alpha)dx^2 + \exp(2\mu)(dy^2 + dz^2) - dt^2$

where $\alpha = \alpha(t)$ and $\mu = \mu(t)$

Now, the energy-momentum tensor for the string dust with a magnetic field along the direction of the string, i.e. the x-direction is given by

(2.2) $T_\beta^\gamma + E_\beta^\gamma = \rho u_\beta u^\gamma - s x_\beta x^\gamma + \frac{1}{4\pi}(F_\beta^\gamma F_\nu^\gamma - \frac{1}{4} F_{\nu\eta}F^{\nu\eta}\delta_\beta^\gamma)$

where in above T_β^γ is the stress-energy tensor for a string dust system, E_β^γ is that for the magnetic field and $F_{\nu\eta}$ is the electromagnetic field tensor. The other terms have already been explained in the previosus section. In the co-moving co-ordinate system $u^\beta = \delta_4^\beta$ and

(2.3) $T_4^4 = -\rho.$ $T_1^1 = -s,$ $T_2^2 = T_3^3 = 0 = T_\beta^\gamma$ (for $\beta \neq \gamma$)

Again, as the magnetic field is in the x-direction, F_{23} is the only non-zero part of the electromagnetic field tensor. Maxwell equation $F_{[\beta\gamma,\alpha]} = 0$ and $(F^{\beta\gamma}. (-g)^{\frac{1}{2}}) = 0$, now provide the result.

(2.4) $F_{23} = H$

H being a constant quantity. So, the components of stress energy tensor for the electromagnetic field are

(2.5) $E_4^4 = E_1^1 = -E_2^2 = -E_3^3 = -\frac{H^2}{8\pi}\exp(-4\mu)$

In natural units $8\pi G = 1$ so that the non-zero components of Einstein field equations

(2.6) $R_\beta^\gamma - \frac{1}{2}\delta_\beta^\gamma R = -(T_\beta^\gamma + E_\beta^\gamma)$

are

(2.7) $2\dot{\alpha}\dot{\mu} + \dot{\mu}^2 = \rho + \frac{H^2}{8\pi} e^{-4\mu}$

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(2.8) $2\ddot{\mu} + 3\dot{\mu}^2 = s + \frac{H^2}{8\pi} e^{-4\mu}$

(2.9) $\ddot{\alpha} + \dot{\alpha}^2 + \ddot{\mu} + \dot{\mu}^2 + \dot{\alpha}\dot{\mu} = -\frac{H^2}{8\pi} e^{-4\mu}$

The scalar of expansion ɵ, proper volume R^3, and scalar of shear tensor σ^2 are

(2.10) $\theta = u^{\beta}_{\beta} = \dot{\alpha} + 2\dot{\mu} = 3\frac{\dot{R}}{R}$

(2.11) $R^3 = \exp(\alpha + 2\mu)$

(2.12) $\sigma^2 = \sigma_{\beta\gamma}\cdot\sigma^{\beta\gamma} = \dot{\alpha}^2 + 2\dot{\mu}^2 - \frac{1}{3}\theta^2$

where

(2.13) $\sigma_{\beta\gamma} = \frac{1}{2}[u_{\beta\gamma} + u_{\gamma\beta} + u_{\beta}u^{\nu}u_{\gamma\nu} + u_{\gamma}u^{\nu}u_{\beta\nu}] - \frac{1}{3}\theta(g_{\beta\gamma} + u_{\beta}u_{\gamma})$

Now, one can directly obtain the Ray Chaudhuri's equation (Ray Chaudhuri [21]) from the above set of field equations (2.7) to (2.9) and using (2.10) and (2.12) as

(2.14) $\theta = \frac{1}{3}\theta^2 - 2\sigma^2 = \frac{1}{2}\cdot\rho_p - \frac{H^2}{8\pi} e^{-4\mu}$

where

(2.15) $R_{\beta\gamma}u^{\beta}u^{\gamma} = -\frac{\rho_p}{2} - \frac{H^2}{8\pi} e^{-4\mu}$

Now in view of all the three (strong, weak and dominant) energy conditions (Wald [29]), one finds $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of s is unrestricted. It may take values positive, negative or zero as well. This implies in view of (2.14) that even the existence of the strings is unable to halt the collapse. From the above energy conditions, we find that s might even take the negative value and therefore Einstein's equation (2.6) with $s < 0$, is the equation for an anisotropic fluid with pressure different from zero along the direction of x^{β}.

3. SOLUTION OF THE FIELD EQUATIONS: -

We have three equations (2.7) – (2.9) in four unknowns α, μ, ρ and thus the system is indeterminate. To make the system determinate, we require one more relation. For this we use different assumptions. Here we assume a relation between α and μ as follows

$\alpha = f(\mu)$

Case I: - Here we choose f to be a constant function say

$f(\mu) = \text{constant} = A$ (say)

In this case equation (2.9) reduces to

(3.1) $\ddot{\mu} + \dot{\mu}^2 = - \frac{H^2}{8\pi} e^{-4\mu}$

Equation (3.3.1) can be put as an integral equation

(3.2) $\int d(\dot{\mu}^2 e^{2\mu}) = - \frac{H^2}{4\pi} e^{-2\mu} d_\mu + A$

where A is constant of integration. So we get

(3.3) $\dot{\mu}^2 = A\ e^{-2\mu} + \frac{H^2}{8\pi} e^{-4\mu}$

which can further be cast as an integral form as

(3.4) $\int \frac{e^{2\mu} d\mu}{\left[Ae^{2\mu} + \frac{H^2}{8\pi}\right]^{\frac{1}{2}}} = \pm (t - t_0)$

where t_0 is another integration constant. Integrating (3.4) we get

(3.5) $e^{2\mu} = - \frac{H^2}{8\pi A} + A\,(t - t_0)^2$

ρ and s can now be found from (2.7) and (2.8) respectively as

(3.6) $\rho = \frac{A}{\left[A\ (t-t_0\ )^2 - \frac{H^2}{8\pi A}\right]}$

(3.7) $s = \frac{\left[A^2\ (t-t_0\ )^2 - 3\cdot\frac{H^2}{8\pi}\right]}{\left[A\ (t-t_0\ )^2 - \frac{H^2}{8\pi A}\right]^2}$

and

(3.8) $\rho_p = \rho - s = \frac{\left(\frac{H^2}{4\pi}\right)}{\left[A\ (t-t_0\ )^2 - \frac{H^2}{8\pi A}\right]^2}$

Finally, expansion scalar ɵ, the proper volume R^3, and shear scalar σ can be obtained from (2.10) to (2.13) respectively as

(3.9) $\theta = \frac{2A(t-t_0\ )}{R^3}$

(3.10) $R^3 = A\cdot(t - t_0\ )^2 - \frac{H^2}{8\pi A}$

(3.11) $\sigma^2 = \frac{1}{6}\left[\frac{2A(t-t_0\ )}{R^3}\right]^2$

From the above solutions we observe that at the initial epoch.

$$(t - t_0\ )^2 = \frac{H^2}{8\pi A^2}$$

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the string model starts with an initial singularity $R^3 \to 0$. while ρ, ρ_p, s, ϴ, σ^2 etc. diverge This is a line singularity, since exp(2α) ⟶ 1 and exp(2μ) ⟶ 0. At a later instant when $(t - t_0)^2 = \frac{3H^2}{8\pi A}$ we have $s_0 = 0$ and ρ = ρ_p. So at this epoch string vanish and we are left with a dust filled universe with a magnetic field. At this stage

$$(3.12) \quad \begin{cases} \rho = \frac{4\pi A^2}{H^2} \\ R^3 = \frac{H^2}{4\pi A} \\ \theta = \frac{2}{H}(6\pi)^{\frac{1}{2}} \\ \quad and \\ \sigma^2 = \frac{2\pi A^2}{3H^2} \end{cases}$$

i.e. all these parameters are of finite magnitude. In this solution matter is directly related with the magnetic field as is noted in (3.8). When the magnetic field is absent, the matter is also absent and the solution reduces to that of pure geometric string distribution.

Case II: - Here we choose f(μ) to be of the form

$$(3.13) \quad f(\mu) = D\mu^n$$

$$\Rightarrow \dot{f}(\mu) = Dn\mu^{n-1}\dot{\mu}$$

$$\Rightarrow \ddot{f}(\mu) = Dn(n-1)\mu^{n-2}\dot{\mu}^2 + Dn\mu^{n-1}\ddot{\mu}$$

By use of (3.13) equation (2.9) gives

$$(3.14) \quad \ddot{f}(\mu) + \left(\dot{f}(\mu)\right)^2 + \ddot{\mu} + \dot{\mu}^2 + \dot{f}(\mu)\dot{\mu} = \frac{H^2}{8\pi} e^{-4\mu}$$

$$\Rightarrow Dn(n-1)\mu^{n-2}\dot{\mu}^2 + \mu^{n-1}\ddot{\mu} + (Dn\mu^{n-1}\dot{\mu})^2 + \ddot{\mu} + \dot{\mu}^2 + Dn\mu^{n-1}\dot{\mu}^2$$

$$= \frac{H^2}{8\pi} e^{-4\mu}$$

$$\Rightarrow (1 + Dn\mu^{n-1})\ \ddot{\mu} + [D^2n^2(\mu^{n-1})^2 + Dn(n-1)\mu^{n-2} + 1 + Dn\mu^{n-1}]\ \dot{\mu}^2$$

$$= \frac{H^2}{8\pi} e^{-4\mu}$$

To avoid mathematical complexity, we take n = 1 so that we have

$$(3.15) \quad f(\mu) = D\mu$$

Then equation (3.14) provides

$$(3.16) \quad (1 + D)\ddot{\mu} + (D^2 + D + 1)\ \dot{\mu}^2 = \frac{H^2}{8\pi} e^{-4\mu}$$

Equation (3.16) can be put as an integral equation

(3.17) $$\int d\left[\dot{\mu}^2 exp\left(2\left(\frac{D^2+D+1}{D+1}\right)\mu\right)\right] = -\frac{H^2}{4\pi(D+1)}\int exp\left(2\left(\frac{D^2-D-1}{D+1}\right)\mu\right)d\mu + C$$

where C is constant of integration. So, we have

(3.18) $$\dot{\mu}^2 = C\ exp\left[\left(-2\left(\frac{D^2+D+1}{D+1}\right)\mu\right)\right] - \frac{H^2}{8\pi(D^2-D-1)}\exp(-4\mu)$$

which can again be written as an integral form as

(3.19) $$\int \frac{e^{2\mu}d\mu}{\left[C\ exp\left[\left(-2\left(\frac{D^2+D+1}{D+1}\right)\mu\right)\right]-\frac{H^2}{8\pi(D^2-D-1)}\right]^{\frac{1}{2}}} = \pm(t-t_0)$$

where t_0 is another constant of integration. To solve (3.18) we choose D such that

(3.20) $$(D^2-D-1) = 2(D+1)$$

$$\Rightarrow D^2 - 3(D+1) = 0$$

which is quadratic equation in D.

Its solution is

(3.21) $$D = \frac{3\pm\sqrt{9+12}}{2}$$

or $$D = \frac{3\pm\sqrt{21}}{2}$$

With above value of D, equation (3.19) can at once be integrated to give

(3.22) $$e^{2\mu} = \left[\frac{8\pi C}{H^2}(5\pm\sqrt{21}) - \frac{H^2(t-t_0)^2}{2\pi(5\pm\sqrt{21})}\right]^{\frac{1}{2}}$$

From (3.22) it is clear that arbitrary constant C must be positive in this case. So, we replace C by L^2 in the following. The other parameters can be found as before

(3.23) $$\rho = \frac{\frac{(9+2\sqrt{21})}{(5+\sqrt{21})^2}\frac{H^2}{16\pi^2}(t-t_0)^2-(5+\sqrt{21})L^2}{\left[\frac{8\pi L^2}{H^2}(5+\sqrt{21})-\frac{H^2(t-t_0)^2}{2\pi(5+\sqrt{21})}\right]^2}$$

From energy conditions, $\rho > 0$ which demands positive sign before $\sqrt{21}$ in equation (3.22). With this choice other parameters are found explicitly as follows

(3.24) $$s = \frac{(9+\sqrt{21})L^2 - \frac{(4+\sqrt{21})}{(5+\sqrt{21})^2}\cdot\frac{H^2}{16\pi^2}(t-t_0)^2}{\left[\frac{8\pi L^2(5+\sqrt{21})}{H^2}-\frac{H^2(t-t_0)^2}{2\pi(5+\sqrt{21})}\right]^2}$$

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(3.25) $$\rho_p = \frac{4L^2 + \frac{H^2(t-t_0)^2}{16\pi^2(5+\sqrt{21})}}{\left[\frac{8\pi L^2(5+\sqrt{21})}{H^2} - \frac{H^2(t-t_0)^2}{2\pi(5+\sqrt{21})}\right]^2}$$

(3.26) $$R^3 = \left[\frac{8\pi L^2(5+\sqrt{21})}{H^2} - \frac{H^2(t-t_0)^2}{2\pi(5+\sqrt{21})}\right] \times \frac{(7+\sqrt{21})}{8}$$

(3.27) $$\theta = -\left(\frac{7+\sqrt{21}}{5+\sqrt{21}}\right)\frac{H^2}{8\pi}\left[\frac{(t-t_0)}{\frac{8\pi L^2(5+\sqrt{21})}{H^2} - \frac{H^2(t-t_0)^2}{2\pi(5+\sqrt{21})}}\right]$$

(3.28) $$\sigma^2 = \frac{7+\sqrt{21}}{(5+\sqrt{21})^2}\left(\frac{H^2}{48\pi^2}\right)\left[\frac{(t-t_0)}{\frac{8\pi L^2(5+\sqrt{21})}{H^2} - \frac{H^2(t-t_0)^2}{2\pi(5+\sqrt{21})}}\right]$$

Now it is clear from (3.22) that

(3.29) $$T^2 < \frac{16\pi^2 L^2}{H^4}(5+\sqrt{21})^2$$

where T = $t_0 - t$. When $t < t_0$ we have T > 0 and clearly from equation (3.27)

$\theta > 0$

Hence, we have the expanding model.

Case III: - When H = 0 (i.e. when magnetic field is absent). Then assuming (3.15) equation (2.9) produces

(3.30) $$(D+1)\ddot{\mu} + (D^2+D+1)\dot{\mu}^2 = 0$$

or $$\ddot{\mu} + \frac{D^2+D+1}{D+1}\dot{\mu}^2 = 0$$

This equation can be put as an integral equation

(3.31) $$\int d\left[\dot{\mu}^2 exp\left(2\left(\frac{D^2+D+1}{D+1}\right)\mu\right)\right] = F$$

where F is integrating constant. Hence, we get

(3.32) $$\dot{\mu}^2 = F\left[exp\left(-2\left(\frac{D^2+D+1}{D+1}\right)\mu\right)\right]$$

which further provides

(3.33) $$\int \frac{e^{2\mu}d\mu}{F\left[exp\left(-2\left(\frac{D^2-D-1}{D+1}\right)\mu\right)\right]} = \pm(t - t_0)$$

where t_0 is another integrating constant.

This differential equation can be integrated on give solution which is not the special case of that given earlier.

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(3.34) $e^{2\mu} = [F(t - t_0)]^{\frac{2(D+1)}{(D^2+D+1)}}$

where F is a constant.

Therefore

(3.35) $e^{2\alpha} = [F(t - t_0)]^{\frac{2D(D+1)}{(D^2+D+1)}}$

The physical parameters are found to be

(3.36) $R^3 = [F(t - t_0)]^{\frac{(D^2+3D+2)}{(D^2+D+1)}}$

(3.37) $\rho = \frac{(2D+1)}{(t-t_0)^2}$

(3.38) $s = \frac{1}{(t-t_0)^2}$

(3.39) $\rho_p = \frac{2D}{(t-t_0)^2}$

(3.40) $\theta = \frac{(D+2)}{(t-t_0)^2}$

(3.41) $\sigma^2 = \frac{2}{3}\frac{(D-1)^2}{(t-t_0)^2}$

Here we see that at the initial epoch, $t = t_0$, $R^3 \to 0$, while ρ, s, ρ_p, θ, σ^2, all diverge, and exp(2α), exp(2μ) → 0 - the point singularity. This is the starting point of the string model. Again, at a later stage when t→∝, R→∝ but all other physical parameters become insignificant. Further we see that for a pure geometric string ($\rho_p = 0$) model, we have to take D = 0. For this case the universe starts with string and ends up at a stage when the massive strings themselves disappear without any remnant.

Again if we consider T = $t - t_0$ and $t > t_0$ then from (3.40) if we take

(i) D > -2 then we get θ to be positive which means that the model is expanding.

(ii) D < -2 then θ = - ve which => contracting model.

(iii) D = - 2 then θ = 0 which => neither expansion nor contraction in the model.

Further if we take T = $t - t_0$ and $t < t_0$ then from (3.40) if we take

(a) D > -2 then θ = -ve which => our model is contracting.

(b) D < -2 then θ = +ve which provides expanding model.

(c) D = -2 then θ = 0 which means our model neither contracts nor expands.

4. CONCLUSION AND DISCUSSION: -

In the present work we have taken a judicious relation between metric potential α and μ in the form of α = f(μ). Our work includes work by Letelier and other researchers in this field by different choice of f(μ) and suitable adjustment of constants. Further we have used a source in absence and presence of magnetic field and have obtained solutions in different cases e.g. $f(\mu) = D\mu^n$. which gives f(μ) = Dμ for n=1. For specific value of D, we see that our model starts from a string dominated era but after instant string vanish and the model converts into particle dominated. In our investigation gravitational field is coupled to magnetic field so that when magnetic field is absent (i.e. H=0), our system reduces to pure geometric string.

Further at initial epoch t = to, we find that $R^3 \to 0$, while ρ, s, ρ_p, ɵ, σ^2 all diverge and exp (2α), exp(2μ) →0 the point of singularity. This is the starting point of the string model. Also, afterwards at a later stage, when t → ∞, R → ∞, but all other parameters like expansion ɵ, shear σ s, ρ_p, become insignificant. Again, we find that when ($\rho_p = 0$) is pure geometric string model, we have to choose D=0. Also, in this case the universe begins with string and finishes at a stage, where massive strings themself disappear without any remnant.

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Vol. 15 Issue 02, February -2025,
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