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New Ideas on Zero Private Proximate Dominating Set

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Abstract

Let G be a connected graph that is non-trivial. If a set $S \subseteq V$ is dominating and each vertex u in S has no privatized neighbor in $V - S$, then the set is considered a Zero Private proximate Dominating Set. The Zero Private proximal Dominating number, represented by $\gamma_{zpp}(G)$, is the minimal cardinality of a Zero Private proximate Dominating Set. The related set is referred to as the $\gamma_{zpp} -$ set of G. In this work, we derive the value of a new parameter, Zero Private Proximate Dominating Number, for a few known graphs.

Keywords: Connected graph, Zero Private Proximate Dominating set, Path graph, Cycle graph, Wheel graph, complete bipartite graph, complete graph, Star graph.

1. Introduction

Graph theory is a fascinating and fundamental branch of mathematics that deals with the study of graphs, which are mathematical structures used to model relationships and connections between objects. A graph, in this context, is not a visual representation but a set of nodes (vertices) connected by edges (lines or arcs).Graphs can be applied to a wide range of real-world problems and scenarios, making graph theory a versatile and powerful tool in various fields, including computer science, biology, social sciences, transportation, and many others. Graphs are used to represent and analyze networks, relationships, and dependencies in a structured and abstract way.

The history of graph theory dates back to the 18th century when the Swiss mathematician Leonhard Euler introduced the famous Seven Bridges of Königsberg problem, which is considered the starting point of graph theory. Since then, graph theory has grown into a rich and diverse field with a wide range of concepts, algorithms, and applications.

Some key concepts in graph theory include directed and undirected graphs, cycles, paths, connectivity, trees, and various types of special graphs like bipartite graphs, planar graphs, and more. Graph algorithms are used to solve problems such as finding the shortest path, identifying connected components, and optimizing network flows.

Graph theory has a profound impact on computer science, particularly in data structures and algorithms. It plays a crucial role in areas like social network analysis, recommendation systems, computer networking, and routing algorithms. In addition to its practical applications, graph theory also has elegant mathematical properties and offers intriguing puzzles and theorems, making it a subject of great interest for mathematicians and researchers.

Domination theory is a subfield of graph theory that deals with the concept of domination in graphs. Domination is a fundamental and important concept in graph theory because it has numerous applications in real-world scenarios, such as network design, facility location, and resource allocation.

In graph theory, a domination set, or dominating set, is a subset of the vertices of a graph that covers or controls the entire graph. More formally, a subset S of the vertices of a graph G is called a dominating set if, for every vertex not in S, there is at least one vertex in S adjacent to it. The process of finding the smallest dominating set in a graph is known as the Minimum Dominating Set problem. Domination theory is concerned with various aspects of dominating sets, including: Dominating Number: The minimum number of vertices required in a dominating set is known as the dominating number of a graph. It is often denoted as $\gamma(G)$.

Total Dominating Number: In some applications, it is important to consider not only the vertices covered by the dominating set but also their adjacent edges. The total dominating number, denoted as $\gamma_t(G)$, takes into account both vertices and edges.

Dominating Sets in Special Graphs: Researchers have explored domination theory in various types of graphs, including trees, grids, and planar graphs. Understanding domination properties in these specific graph classes has practical implications in network design and optimization.

Domination theory has applications in various fields, including wireless communication, sensor networks, social network analysis, and facility location. For example, in wireless sensor networks, dominating sets can be used to optimize the placement of sensors to monitor an area effectively while conserving energy. In social networks, they can help identify influential individuals or control the spread of information.

In summary, domination theory in graph theory focuses on understanding and characterizing dominating sets in graphs, with applications in network design, optimization, and various other real-world scenarios. It provides valuable insights into the structure and behavior of graphs in practical applications.

Conducting a literature survey on domination theory in graph theory involves reviewing a wide range of academic papers, books, and research articles to understand the key concepts, recent developments, and applications in this field. Below, I provide an outline of the topics and some key references that can serve as a starting point for your literature survey. Keep in mind that this is not an exhaustive list, and there may be many more relevant sources to explore.

Begin by exploring foundational concepts in domination theory, including the definition of dominating sets, the dominating number, and various types of domination (e.g., total domination, connected domination).Study algorithms and approaches for finding minimum dominating sets in graphs, including exact algorithms, approximation algorithms, and heuristic methods. Investigate domination theory in specific types of graphs, such as trees, grids, planar graphs, and more. Understand how domination properties differ in these graph classes. Explore the computational complexity of domination problems in different graph classes. Understand the NP-hardness and parameterized complexity of domination problems.

Research variations and extensions of domination theory, such as fractional domination, secure domination, and power domination, and their applications. Investigate how domination theory is applied in various real-world scenarios, including network design, wireless communication, sensor placement, social network analysis, facility location, and more. Stay updated on recent research papers and trends in domination theory.

The concept of the private dominating set was first presented by Bollabas and Cockaye. They also demonstrated that every network with no isolated vertices has a private minimum dominating set. This motivates us to create zero private proximate dominance number of a graph, another additional parameter. An attempt has been made to obtain specific characterizations of this parameter, in this study, Zero Private proximate Dominating Set:

In this paper, we define the new parameter called Zero Private proximate Dominating number and obtain its value for certain known graphs.

2. Main Results

2.1. Definition

Let G be a connected graph that is non-trivial. If a set $S \subseteq V$ is dominating and each vertex u in S has no privatized neighbor in $V - S$, then the set is considered a Zero Private proximate Dominating Set. The Zero Private proximal Dominating number, represented by $\gamma_{zpp}(G)$, is the minimal cardinality of a Zero Private proximate Dominating Set. The related set is referred to as the $\gamma_{zpp} -$ set of G.

Hereafter we denote a Zero Private proximate Dominating Set as γ_{zpp}-set.

2.2. Example

For the graph G given in Figure 2.1, one can see that $\{1,3,5\}, \{2,4,6,7\}$ are minimal γ_{zpp}-sets and thus $\gamma_{zpp}(G) = 3$.

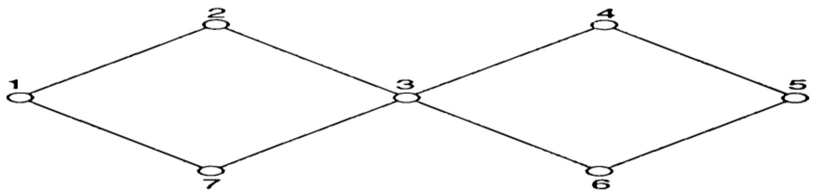

Figure 2.1:

Figure 2.2:

2.3. Remark

From the Definition 2.1 one can observe that a γ_{zpp}-set need not be a minimal dominating set. In fact for the graph given in Figure 2.2, it is easy to verify that $\{1,3,6,8\}$ is a γ_{zpp} $-$set of G but not a minimal dominating set.

The following lemma will be useful in the subsequent sections.

3. Theorems

The following are true:

(i) For any graph $G, \gamma(G) \leq \gamma_{zpp}(G)$.

(ii) $\gamma_{zpp}(P_k) = \left\lceil \frac{k+1}{2} \right\rceil$.

(iii) $\gamma_{zpp}(C_k) = \left\lceil \frac{k}{2} \right\rceil$.

(iv) $\gamma_{zpp}(K_s) = 2$ for $s \geq 2$.

(v) $\gamma_{zpp}(K_{l,n}) = mi\{l,n\}$ for $l \geq 2$ and $n \geq 2$.

(vi) $\gamma_{zpp}(K_{1,s-1}) = s-1$ for $s \geq 3$.

(vii) $\gamma_{zpp}(K_s) = \gamma_{zpp}(K_s - v)$ for $s \geq 3$ and any vertex v in K_s.

(viii) $\gamma_{zpp}(K_s) = \gamma_{zpp}(K_s - e)$ for $s \geq 3$ and any vertex e in K_s.

(ix) $\gamma_{zpp}(W_t) = 2$ for $3 \leq t \leq 5$ and $\gamma_{zpp}(W_t) = \left\lceil \frac{t+2}{3} \right\rceil$ for $t \geq 6$ where W_t is a wheel with t vertices .

(x) $\gamma_{zpp}(Mo\ K_1) = |V(M)| + \gamma(M)$.

(xi) For any graph G, $2 \leq \gamma_{zpp}(G) \leq p-1$ for $p \geq 2$.

Proof.

(i) Obvious by the definition 2.1.

(ii) Let$\{v_1, v_2, v_3, \dots, v_k\}$ be the vertices of the path P_k.Case (a) when k is odd. In this case $\{v_1, v_3, v_{k-3}, v_{k-1}, v_k\}$ is a minimum $zpp-$ set. Hence, $\gamma_{zpp}(P_k) = \frac{k}{2} + 1 = \left\lceil \frac{k}{2} + \frac{1}{2} \right\rceil = \left\lceil \frac{k+1}{2} \right\rceil$. In both cases, $\gamma_{zpp}(P_k) = \left\lceil \frac{k+1}{2} \right\rceil$.

(iii) Similar to that of (ii).

(iv) For ay vertex x in K_s, $\{x\}$ is a γ-set of K_s and x has $s-1$ private neighbours in $K_s - x$. Now every vertex in $K_s - \{x,y\}$ is adjacent to both x and y. Therefore $\{x,y\}$ is a γ_{zpp}-set of K_s. Hence, $\gamma_{zpp}(K_s) = 2$.

(v) Let (P,Q) be a partition of the vertex set of the bipartite graph $K_{l,n}$ with $l \geq 2$ and $n \geq 2$. One can see that both P and Q are the only minimal $zpp-$ sets. Hence, $\gamma_{zpp}(K_{l,n}) = \min\{|P|,|Q|\} = min\{l,n\}$.

(vi) Since leaving any pendant vertex of $K_{1,s-1}$will make it as a private vertices must be included in the γ_{zpp} $-$ set of $K_{1,s-1}$. Hence $\gamma_{zpp}(K_{1,s-1}) = s-1$.

(vii) For any vertex v in K_s, $K_s - v$ is also a complete graph. By (iv) $\gamma_{zpp}(K_s) = \gamma_{zpp}(K_s - v) = 2$

(viii) Let $e = uv$ be a edge in K_s. Then u and v are adjacent to every vertex in $K_s - e$. By (iv) $\gamma_{zpp}(K_s) = \gamma_{zpp}(K_s - e) = 2$

(ix) It is easy to see that $\gamma_{zpp}(W_3) = \gamma_{zpp}(W_4) = \gamma_{zpp}(W_5) = 2$.

For $p \geq 6$, note that $W_t = C_{t-1} + K_1$.

From this it follows that a $\gamma-$set of C_{t-1} together with $V(K_1)$ is γ_{zpp} $-$ set of W_t. Therefore $\gamma_{zpp}(W_t) = \gamma(C_{t-1}) + |V(K_1)| = \left\lceil \frac{t-1}{3} \right\rceil + 1 = \left\lceil \frac{t+2}{3} \right\rceil$.

(x) Any γ-set of M together with all the pendant vertices in $Mo\ K_1$ from a γ_{zpp}- set of $Mo\ K_1$. Hence $\gamma_{zpp}(Mo\ K_1) = |V(M)| + \gamma(M)$.

(xi) Let G be a graph with p vertices. Then G is a spanning subgraph of K_p and $\gamma_{zpp}(G) \leq \gamma(G)$. from (iv), the result follows.

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