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A UNIQUE COMMON FIXED POINT THEOREM IN HILBERT SPACE FOR SELF MAPPINGS

Mujahida Sayyed*, Farkhunda Sayyed and Shoyeb Ali Sayyed*****
***Assistant Professor, College of Agriculture, Jnkvv, Ganjbasoda Distt Vidisha M.P. India.**
**** Professor, SAGE University,** Indore(M.P.)India
*** Principal, Royal College of Technology Indore(M.P.)India

ABSTRACT

In this present review article, I boosted the developments of prominent educators as well as analyzers. Using I- Scheme and self mappings along with closed convex subset in Hilbert space for gathering a unique common fixed point . For getting hold of we did some tempering in old survey. Our desired result of the theorem is governing by a great number of experts.

KEY WORDS AND PHRASES : Closed Convex Subset , Functional Inequality, Hilbert Space, Ishikawa Iteration, Self Mappings, Unique Common Fixed Point

AMS(2010) SUBJECT CLASSIFICATIONS*:* Primary 47H10 Secondary 54H25

1. INTRODUCTION AND PRELIMINARY

Availing oneself of Rhoades [14,15] opinion Naimpally and Singh[8] extended it by using contraction condition , and sequel Sayyed and Badshah [17,18,19,20] forwarded it. Again Imdad and Jawed [4]observed that the general form of his theorem remains true in metric spaces. In same pattern we gave result on nonlinear contraction with gaining a fixed point as well as Sayyed et.al.[21,22] examined it for self maps. For consequently started with Ciric [1], Das and Gupta [2], Yadav et,al [25], Veerapandi and Kumar [24],Rao et.al.[13] , Patel and Sharma [12].Nigam et.al. [9], Park [11],Dixit and Bhargav [3], Koparde and Waghmode[5,6], Modi and Gupta[7], Sharma et.al [23], Sangar andWaghmode[16] and Pandhare and Waghmode[10].

As follows to explain some of the changes made in-place:

(i) In the Ishikawa scheme $\{\mu_{2n}\}$, $\{\omega_{2n}\}$ satisfies $0 \leq \mu_{2n}$, $\omega_{2n} \leq 1$, $0 \leq \mu_{2n}$, $\omega_{2n} \leq 1$, $\forall$ n,
$\lim \omega_{2n} = 0$ as $n \to \infty$ and $\sum \mu_{2n} \omega_{2n} = \infty$.

(ii) $\lim_{n\to\infty} \mu_{2n} = \mu_0 > 0$

(iii) $\lim_{n\to\infty} \omega = \omega_0 > 1$.

Let C* be a non empts subset of B*, where B* is a Banach space as well as G and G* be two mappings in C* to C* The iteration scheme, called I- Scheme, defined as follows:

$r_0 \in C^*$

.........(1.1)

$s_{2n} = \omega_{2n} Gr_{2n} + (1- \omega_{2n}) r_{2n} \quad n \geq 0$

$r_{2n+1} = (1- \mu_{2n}) r_{2n} + \mu_{2n} G^* s_{2n} \quad n \geq 0$

.... (1.2)

$s_{2n+1} = \omega_{2n+1} Gr_{2n+1} + (1- \omega_{2n+1}) r_{2n+1} \quad n \geq 0$

$r_{2n+1} = (1- \mu_{2n+1}) r_{2n+1} + \mu_{2n+1} G^* s_{2n+1} \quad n \geq 0$

......(1.3)

Its well known that B* is H* if and onls if its norms satisfies the parallelogram law, i.e. $\forall$ r, s $\in B*$

$$||r+s||^2 + ||r-s||^2 = 2\,||r||^2 + 2||s||^2$$

..........(1.4)

which implies, $||r+s||^2 \leq 2\,||r||^2 + 2||s||^2$

......(1.5)

2.MAIN RESULT

THEOREM 2.1 : Taking G and G* as two self mapping in a Hilbert space denoted by H* with a closed convex subset abbreviated by C*in Hilbert space H* also satisfying

$$||Gr-G^*s||^2 \leq \Phi \max \{||r-s||^2 , \frac{||s-G*s||^2[1+||r-Gr||^2]}{1+||r-s||^2} , \frac{[1+||s-G*s||^2]||r-Gr||^2}{1+||r-s||^2} , [||r-Gr||^2 + ||s-G^*s||^2] , [||r-G^*s||^2 + ||s - Gr||^2] \}$$

...... (2.1)

Taking Φ is arbitrary positive with $0 \leq 4\Phi \leq 1$ and if $\exists$ r_0 such that the I- scheme for G and G* defined by (1.2) and (1.3), converges to a point z*, then z* is a common point of G and G*.

PROOF:

From equation (1.2) , describe $r_{2n+1} - r_{2n} = \mu_{2n} (G^* s_{2n} - m_{2n})$

Since $r_{2n} \to u$, $\| r_{2n+1} - r_{2n} \| \to \infty$

Since $\{ \mu_{2n} \}$ is bounded away from zero, $\|G^*s_{2n} - r_{2n}\| \to 0$ as $n \to \infty$.

It follows that $\|u - G^*s_{2n}\| \to 0$ as $n \to \infty$.

Since G and G* satisfy (2.1) we have

$$\|Gr_{2n}-G^*s_{2n}\|^2 \leq \Phi \max\Big\{ \|r_{2n} - s_{2n}\|^2 , \frac{\|s_{2n}-G^*s_{2n}\|^2[1+\|r_{2n}-Gr_{2n}\|^2]}{1+\|r_{2n}-s_{2n}\|^2}, \frac{[1+\|s_{2n}-G^*s_{2n}\|^2]\|r_{2n}-Gr_{2n}\|^2}{1+\|r_{2n}-s_{2n}\|^2}, [\|r_{2n}-Gr_{2n}\|^2 + \|s_{2n}-G^*s_{2n}\|^2], [\|r_{2n}-G^*s_{2n}\|^2 + \|s_{2n} - Gr_{2n}\|^2] \Big\}$$

Now, $\|s_{2n} - r_{2n}\|^2 = \| \omega_{2n} Gr_{2n} + (1-\omega_{2n}) r_{2n} - r_{2n} \|^2$
$= \| \omega_{2n} Gr_{2n} + r_{2n} - \omega_{2n} r_{2n} - c \|^2$
$= \| \omega_{2n} (Gr_{2n} - r_{2n}) \|^2$
$= \omega^2_{2n} \| (Gr_{2n} + G^*s_{2n}) + (G^*s_{2n} - r_{2n}) \|^2$
$\leq 2 \|Gr_{2n} - G^*s_{2n}\|^2 + 2 \| G^*s_{2n} - r_{2n}\|^2$ ....(2.3)

and
$\|s_{2n} - G^*s_{2n}\|^2 = \| \omega_{2n} Gr_{2n} + (1-\omega_{2n}) r_{2n} - G^*s_{2n}\|^2$
$= \| \omega_{2n} Gr_{2n} + (1-\omega_{2n}) r_{2n} - G^*s_{2n} + \omega_{2n} G^*s_{2n} - \omega s_{2n} G^*s_{2n} \|^2$
$= \| \omega_{2n} (Gr_{2n} - G^*s_{2n}) + (1-\omega_{2n}) (r_{2n} - G^*s_{2n})\|^2$
$\leq 2 \omega^2_{2n} \| Gr_{2n} - G^*s_{2n}\|^2 + 2 (1-\omega_{2n})^2 \| r_{2n} - G^*s_{2n}\|^2$
$\leq 2 \| Gr_{2n} - G^*s_{2n}\|^2 + 2 \| r_{2n} - G^*s_{2n}\|^2$ ....(2.4)

from (2.2), (2.3) , (2.4) can be written as:

$$\|Gr_{2n}-G^*s_{2n}\|^2 \leq \Phi \max\Big\{[2 \|Gr_{2n} - G^*s_{2n}\|^2 + 2 \| G^*s_{2n} - r_{2n}\|^2], \frac{[2\|Gr_{2n}-G^*s_{2n}\|^2 + 2 \|r_{2n}-G^*s_{2n}\|^2][1+2\|r_{2n}-G^*s_{2n}\|^2+2\|G^*s_{2n}-Gr_{2n}\|^2]}{1+2 \|Gr_{2n}-G^*s_{2n}\|^2 + 2\|G^*s_{2n}-r_{2n}\|^2}, \frac{[1+2\|Gr_{2n}-G^*s_{2n}\|^2 +2\|r_{2n}-G^*s_{2n}\|^2][2 \|r_{2n}-G^*s_{2n}\|^2+2\|G^*s_{2n}-Gr_{2n}\|^2)}{1+ 2 \|Gr_{2n}-G^*s_{2n}\|^2 + 2\|G^*s_{2n}-r_{2n}\|^2}, [2\|r_{2n}-G^*s_{2n}\|^2 +2 \|G^*s_{2n}-Gr_{2n}\|^2 + 2 \| Gr_{2n} - G^*s_{2n}\|^2 + 2 \| r_{2n} - G^*s_{2n}\|^2], [\|r_{2n}-G^*s_{2n}\|^2 + 2 \| Gr_{2n} - G^*s_{2n}\|^2 + 2 \| r_{2n} - G^*s_{2n}\|^2] \Big\}$$

Or

$$\|Gr_{2n}-G^*s_{2n}\|^2 \leq \Phi \max\Big\{[2 \|Gr_{2n} - G^*s_{2n}\|^2 + 2 \| G^*s_{2n} - r_{2n}\|^2], [2\|Gr_{2n} - G^*s_{2n}\|^2 + 2 \|r_{2n} - G^*s_{2n}\|^2], [2\|r_{2n}-G^*s_{2n}\|^2 +2 \|G^*s_{2n}-Gr_{2n}\|^2], [4\|r_{2n}-G^*s_{2n}\|^2 + 4\|Gr_{2n}-G^*s_{2n}\|^2], [3\|r_{2n}-G^*s_{2n}\|^2 + 2 \| Gr_{2n} - G^*s_{2n}\|^2]$$

Or

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$||Gr_{2n}-G^*s_{2n}||^2 \leq \Phi\,(2||Gr_{2n}-G^*s_{2n}||^2 +2\,||r_{2n}-G^*s_{2n}||^2$
Or
$(1-2\Phi)\ ||Gr_{2n}-G^*s_{2n}||^2 \leq 2\Phi\,||r_{2n}-G^*s_{2n}||^2$

Or
$||Gr_{2n}-G^*s_{2n}||^2 \leq \frac{2\Phi}{1-2\Phi}||r_{2n}-G^*s_{2n}||^2$
Taking the lim as n→ ∞, we get $||Gr_{2n} - G * s_{2n}||^2 \to 0$. It follows that
$||r_{2n} - Gr_{2n}\ ||^2 \leq ||r_{2n} - G * s_{2n}\ ||^2 + 2||G * s_{2n} - Gr_{2n}\ ||^2 \to 0$
and,
$||z- Gr_{2n}||^2 \leq 2||z - r_{2n}||^2 + 2\ ||r_{2n} - G * s_{2n}\ ||^2 \to 0\ as\ \to \infty.$
If r_{2n} and z satisfy (2.1) we have

$||Gr_{2n}-G^*z||^2 \leq \Phi \max \{||r_{2n} -z\,||^2 , ||z-G^*z||^2 [1+ ||r_{2n}-Gr_{2n}||^2]\ / 1+ [||r_{2n}-z||^2$
$,[1+ ||z-G^*z||^2]\ ||r_{2n}-Gr_{2n}||^2\ / 1+ [||r_{2n}-z||^2$
$, [||r_{2n}-Gr_{2n}||^2 + ||z-G^*z||^2] , [||r_{2n}-G^*z||^2 + ||z - Gr_{2n}||^2]\}$
$||Gr_{2n}-G^*z||^2 \leq \Phi \max \{||r_{2n} -z\,||^2 , [2||\,z- Gr_{2n}||^2 +2||\,Gr_{2n} - G^*z||^2]\ [1+ ||r_{2n}-Gr_{2n}||^2]\ / 1+ [||r_{2n}-z||^2$
$,[\,1+ 2||\,z- Gr_{2n}||^2 +2||\,Gr_{2n} - G^*z||^2\,]\ ||r_{2n}-Gr_{2n}||^2\ / 1+ [||r_{2n}-z||^2$
$, [||r_{2n}-Gr_{2n}||^2 + 2||\,z- Gr_{2n}||^2 +2||\,Gr_{2n} - G^*z||^2\,] , [||r_{2n}-G^*z||^2 + ||z - Gr_{2n}||^2]\,\}$
Taking the lim as n→ ∞, we obtain
$||Gr_{2n} - G * z||^2 \leq 2\Phi\ ||Gr_{2n} - G * z||^2$
$(1-2\Phi)\ ||Gr_{2n} - G * z||^2 \leq 0.$
that is, $||Gr_{2n} - G * z||^2 \to 0$
Finally $||z-G^*z||^2 = ||\,z - Gr_{2n} +Gr_{2n} - G^*z||^2$
$\leq 2||\,z - Gr_{2n}\,||^2 + ||\,Gr_{2n} - G^*z||^2 \to 0$ as n→ ∞,
For that z = G*z
Proceeding in the same , z= Gz.
Clearly by definition G and G* have a common fixed point z.
This complete the proof of theorem.
Assuming G = G* = Z* in previous theorem, we obtain the following corollary :
COROLLARY 2.1: Let H* be a Hilbert space , C* be a closed convex subset of H* and Z* be a self mapping in H* into itself satisfying condition 2.1. Taking Φ is arbitrary positive with $0 \leq 4\Phi \leq 1$

If there exists a point r_0 such that the I-scheme for Z* defined by

$$s_n = \omega_n Z_{rn} + (1- \omega_n)\, r_n,\ n \geq 0$$

$$r_{n+1} = (1- \mu_n)\, r_n + \mu_n Z_{sn}\ ,\ n \geq 0$$

converges to a point p, then p is the fixed point of Z*.

In the I- scheme ,{ μ_n }, { ω_n } satisfy $0 \le \mu_n \le \omega_n \le 1$ for all n.

$\lim_{n\to\infty} \omega_n . \sum \omega_n \mu_n = 0$ Assuming that

(i) $0 \le \omega_n , \mu_n \le 1$, for all n.

(ii) $\lim \omega_n = \omega > 0$,

(iii) $\lim \mu_n = \mu > 1$.

The proof is similar to above Theorem, Hence we omit the details.

CONFLICT OF INTEREST . There is no conflict of interest.

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