

MATHEMATICAL EVALUATION OF ARTERIAL FLOW IN HUMAN BODY

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Abstract

Cardiovascular ailments are a leading cause of increasing mortality worldwide. It's vital to comprehend the connection between these disorders and hemodynamic factors to estimate their prevalence accurately. Simulating blood flow offers a deeper insight into this relationship. This review examines previous simulation studies focused on exploring blood flow dynamics.

Keywords: Blood flow, stenosis, cardiovascular, hemodynamic

Introduction

The cardiovascular system plays a vital role in distributing oxygen and nutrients throughout the body. Consisting primarily of the heart and blood vessels, it is essential for sustaining life. Cardiovascular diseases encompass various pathologies affecting the heart and associated organs, often resulting in fatalities. Hemorheology, the scientific field concerned with the interaction between blood flow and blood vessels, explores these dynamics.

Human blood comprises plasma and blood cells, with red blood cells (RBCs or erythrocytes) being the predominant cell type, influencing fluid behavior due to their semisolid nature. Understanding the material properties of blood vessels and blood flow is crucial for comprehending vascular function.

Experimental research endeavors have delved deeply into characterizing blood flow to better grasp fluid behavior within the human system, which is critical for detecting arterial diseases. Numerical simulation models have become increasingly prevalent in studying blood flow dynamics and turbulence associated with arterial diseases. This study seeks to review these investigations to gain a comprehensive understanding of blood flow dynamics.

Historical Background

Blood flow in stenosed blood vessels has been the subject of extensive research, with various studies focusing on understanding the complex interplay of factors influencing flow characteristics. Srivastava (2002) investigated the impact of stenosis shape and hematocrit on blood flow properties. Analytical results provided insights into flow characteristics in both normal and diseased arteries. Venkateshwarlu and Rao (2004) employed numerical methods to study unsteady blood flow through stenosed arteries with variable viscosity. Their findings highlighted the effects of hematocrit, frequency parameter, and constriction parameters on flow rate, pressure gradient, and wall shear stress. Musad and Khan (2010) developed a model to assess the influence of wall shear stress on blood flow through symmetric stenosed arteries. Their analysis revealed a direct relationship between wall shear stress and stenosis length and height.

Srivastava (2003) introduced a two-layered model for blood flow in stenosed vessels, considering the central layer as a couple stress fluid. The study demonstrated increased flow characteristics magnitude during non-Newtonian fluid behavior compared to Newtonian flow, with the peripheral layer significantly reducing flow resistance. Chakravarty et al. (2004) extended this two-layered model to tapered flexible stenosed arteries, with the central layer represented by a Casson fluid and the peripheral layer as a Newtonian fluid. Their analysis of unsteady flow under pulsatile pressure gradient utilized finite difference schemes. Ponalagusamy (2007) explored a two-layered model for axisymmetric stenosed arteries, assuming both layers as Newtonian fluids. Analytical expressions were derived for slip velocity, flow resistance, core viscosity, and peripheral layer thickness.

Joshi et al. (2009) investigated a composite stenosed artery model, observing increased wall shear stress and flow resistance with higher viscosity in the peripheral layer. This highlighted the importance of considering the peripheral layer in assessing diseased arteries. Sankar and Ismail (2009) compared two different two-fluid models for blood flow in stenosed arteries, demonstrating that the Casson fluid model outperformed the Herschel-Buckley fluid model in terms of plug core radius, pressure drop, and resistance to flow.

Biswas and Chakrabarty (2010) proposed a two-layered model for pulsatile blood flow in stenosed tubes, with the core region assumed to be a Bingham plastic fluid and the peripheral region as a Newtonian fluid. Analytical expressions were derived for various flow parameters using perturbation techniques.

Moayeri and Zendehebudi (2003) made an assumption regarding blood behavior, considering it as a Newtonian fluid within constricted arteries. They delved into the effects of the arterial wall's elastic properties on blood flow characteristics by solving transformed equations related to both the wall and flow. This involved addressing boundary conditions at the interface utilizing the control volume method and the well-known SIMPLER algorithm. Sharma et al. (2004) conducted a study investigating how magnetic fields influence blood flow in elastic arteries, specifically examining both the power law model and the generalized Maxwell model. Their findings suggested a significant impact of magnetic fields on regulating blood flow, particularly notable in hypertensive patients. Voltairas et al. (2005) took a different approach, analyzing arterial blood pressure and flow waveforms by deriving analytical solutions from one-dimensional linear hydrodynamic equations, specifically focusing on anharmonic solutions.

Wang and Parker (2004) utilized the method of characteristics to compute pressure pulse propagation within large arteries. Their study highlighted the importance of considering reflected and absorbed waves, leading to the inference that a complex wave pattern is an indispensable factor in understanding arterial hemodynamics. Pivkin et al. (2005) introduced a comprehensive three-dimensional model to examine pulsatile flow in bifurcated coronary arteries. They explored the effects of both pulsatile flow and dynamic curvature on wall shear stress, offering insights into the complex interplay of factors influencing arterial blood flow.

Stangeby and Ethier (2002) conducted a study that involved a coupled analysis of luminal blood flow and transmural fluid flow. This analysis was achieved by solving the Brinkman model, an extension of the Navier-Stokes equations in porous media, using the Petrov-Galerkin finite element method. They observed an increase in Darcian permeability at the lower side of stenosis, potentially leading to higher deposition of LDL in that region. Kaazempur-Mofrad et al. (2005) compared mass transport and fluid flow results in stenotic arteries, considering both axisymmetric and asymmetric constriction scenarios. They concluded that accurately representing constriction geometries is crucial for diagnosing pathological conditions accurately. Kumar et al. (2005) utilized the finite element method to solve Navier-Stokes equations for a three-dimensional double-branched model of blood flow in the canine aorta with porous effects. They analyzed shear stress at Reynolds number 1000, varying the branch to main aortic flow rate ratio. Their study extensively discussed steady flow, branch flow, and shear stress, comparing their findings with experimental data.

Chakravarty and Sen (2005) presented a mathematical model illustrating the heat and mass transfer response in bifurcated stenotic arteries. They graphically detailed the effects of stenosis on arterial wall motion and

unsteady behavior. Ai and Vafai (2006) developed a model for mass transport in stenotic arterial walls using advection-diffusion equations in a porous medium. Meanwhile, Yang and Vafai (2006) explored the impact of hypertension on macromolecular transport within arteries, discussing filtration velocity and LDL concentration under various clinical conditions.

Khakpou and Vafai (2007) provided an analytical solution for macromolecular transport within arteries, employing Navier-Stokes equations coupled with mass transport equations, while considering the heterogeneous arterial wall. Their model incorporated the Staverman filtration coefficient to account for selective permeability, yielding analytical results in agreement with previous numerical findings. Olgac et al. (2008) simulated LDL mass transport in stenosed coronary arteries, representing the innermost layer, endothelium, with a three-pore model. They discussed the effects of local wall shear stress on LDL concentration.

Overall, these studies have utilized computer simulation methods to gain a deeper understanding of blood flow variables and their implications in stenotic arterial conditions.

Conclusion

The focus on the interaction between vessel walls and blood flow variables has been demonstrated to yield more precise and authentic outcomes in various studies. Utilizing computer simulations has been a common approach in exploring the intricate properties of blood and vessels, shedding light on their complexities. However, the process of gathering data for these simulations has presented significant challenges.

Moreover, accurately estimating the presence and extent of diseased arteries remains a complex phenomenon. Simulation models have emerged as valuable tools in medical diagnosis, offering substantial solutions to address such intricate problems. Through these models, medical professionals can gain

insights into the dynamics of diseased arteries, aiding in more accurate diagnosis and treatment planning. Thus, the integration of simulation techniques into medical practice holds promise for enhancing our understanding and management of cardiovascular diseases and related conditions.

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