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ANALYSIS OF GROUP THEORY THROUGH COMPLEX INTUITIONISTIC FUZZY SETS

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Abstract
This study provides a bridge between complex numbers and intuitionistic fuzzy sets by introducing and exploring the idea of difficult intuitionistic fuzzy subgroups. It proves that there are two new subgroups generated for every complex intuitionistic fuzzy subgroup, each of which has its own special algebraic features. For complicated intuitionistic fuzzy sets, the creation of "level subsets" enables the study of their algebraic properties, demonstrating that these subsets form subgroups and affording insights into their underlying structures. Complex intuitionistic fuzzy subgroups are analyzed under group homomorphisms to reveal how group transitions affect them. In addition, it is shown that a more complete comprehension of subgroup operations is achieved by multiplying two complex intuitionistic fuzzy subgroups. Finally, the study explores the unexpected insights and potential applications found in group theory by investigating complicated inductive fuzzy subgroup' linear composition. In sum, this study adds to our knowledge of intricate intuitionistic fuzzy subgroups, their fascinating algebraic properties, and their profound effect on fuzzy group theory, paving the way for future studies and practical applications in a wide range of mathematical areas.
Keywords:*Intuitionistic Fuzzy Subgroups;Complex Numbers;Algebraic Properties*

1. Introduction

Group theory is a crucial part of algebra, providing a powerful framework to analyze symmetric objects**.** It plays a vital role in classifying symmetries in various fields such as molecular structures, crystals, and genetic codes. Additionally, group theory finds extensive applications in the use of encryption, theories of physics, and geometrical algebra. Alternatively, fuzzy set theory can be utilized addresses ambiguity in human cognitive processes and decision-making, allowing for better problem-solving in everyday life. Intuitionistic fuzzy sets and other extended fuzzy sets have proven valuable in handling uncertain data and multi-criteria decision-making **(Gulzar, et al 2020)**.However, these existing models may not fully capture changes and variations over time, leading to a loss of information **(Sharma, 2011)**.

Complex intuitionistic fuzzy subgroups, a significant algebraic extension of intuitionistic fuzzy subgroups and subrings, are the topic of this work as a means of overcoming these restrictions. **(Atanassov and Vassiliev 2020)** argue that complicated fuzzy-thinking sets' cycle term is important for dealing with intuitionistic fuzzy alternating groups because it appropriately captures cycles in algebraic structures. Different cycles, together with the membership and non-membership functions associated with them, may be systematically found using complicated intuitionistic fuzzy sets. The major impetus for creating and constructing the idea of fuzzy groupings with complicated intuitionistic properties are phase term's distinctive proficiency. Many problems in group theory may now be fully characterized using this novel algebraic structure.

2. Methodology

In this article, we present a novel extension of intuitionistic fuzzy subgroups, introducing the concept of "complex intuitionistic fuzzy subgroups" (CIFSGs). In CIFSGs, the degrees of membership and non-membership are uniquely represented within the unit disk, setting them apart from conventional intuitionistic fuzzy subgroups. These definitions serve as the basis for our further discussions and analysis. The main focus of this work lies in CIFSGs, and we describe their fundamental properties and characteristics, shedding light on their distinct algebraic structure and behavior. Throughout the article, we present related results and theoretical developments that underpin the introduction and exploration of CIFSGs. These results play a pivotal role in our analysis, providing essential insights into the properties and potential applications of CIFSGs. By representing the categories of representation and non-participation throughout the component disk, CIFSGs offer a comprehensive view of their algebraic nature, allowing for a deeper understanding of their unique attributes. We emphasize the significance of this representation in distinguishing CIFSGs from traditional IFSGs and its implications for the study of complex algebraic structures. This systematic approach enables us to unveil the theoretical underpinnings of CIFSGs and their potential applications in various mathematical fields.

3. Preliminaries

Definition 1 :(Atanassov, K., & Vassilev, 2020) For each m in the universe of “discourse P, the intuitionistic fuzzy set (IFS) A is defined as $A = \{(m, \eta_A(m), \hat{\eta}_A(m)) : m \in P\}$, where η_A and $\hat{\eta}_A$ supply the membership and nonmembership values of m from the unit interval, respectively, and $0 \leq \eta_A(m) + \hat{\eta}_A(m) \leq 1$, for any $m \in P$.”

$$\eta_A(mn) \geq \min\{\eta_A(m), \eta_A(n)\},$$
$$\eta_A(m^{-1}) \geq \eta_A(m),$$
$$“\hat{\eta}_A(mn) \leq \max\{\hat{\eta}_A(m), \hat{\eta}_A(n)\},”$$
$$\hat{\eta}_A(m^{-1}) \leq \hat{\eta}_A(m), \text{ for all } m, n \in H.$$

When it comes to object membership and not-membership functions, CIFS A of compact non-empty set P can be seen in a manner that

$$“i = \sqrt{-1}\eta_A(m), \hat{\eta}_A(m), \varphi_A(m), \text{ and } \hat{\varphi}_A(m)”$$

are real valued such that

$$0 \leq \eta_A(m) + \hat{\eta}_A(m) \leq 1 \text{ and } 0 \leq \varphi_A(m) + \hat{\varphi}_A(m) \leq 2\pi.$$

Definition 2:(Alkouri and Salleh, 2013) Two CIFSs of the set P are$A$ and B. Then, we may define “the union of CIFSs A and B as”:

“$A \cup B = \{(m, \theta_{A\cup B}(m), \hat{\theta}_{A\cup B}(m)) : m \in P\}$.”

Where

$$"\theta_{A\cup B}(m)" = "\eta_{A\cup B}(m)e^{i\varphi_{A\cup B}(m)}" = "\max\{\eta_A(m), \eta_B(m)\}e^{i\max\{\varphi_A(m), \varphi_B(m)\}}",$$

and

$$"\hat{\theta}_{A\cup B}(m)" = "\hat{\eta}_{A\cup B}(m)e^{i\hat{\varphi}_{A\cup B}(m)}" = "\min\{\hat{\eta}_A(m), \hat{\eta}_B(m)\}e^{i\min\{\hat{\varphi}_A(m), \hat{\varphi}_B(m)\}}."$$

Here, we focus on CIFSGs and subgroups of CIFSGs that are organized by level. Further, we discovered that a CIFSGs yields a total of two IFSGs. We investigate certain algebraic characteristics of level subsets of CIFS and show that they constitute a subgroup of the CIFSG.

Definition 3: Assume that

$$"A = \{< m, \psi_A(m), \hat{\psi}_A(m)" \quad : m \in H\}$$

a fuzzy set that is based on intuition; an IFS. Then, we may characterize the intuitionistic "fuzzy set (π-IFS) A_π as"

$$"A_{\pi}" = "\{< m, \psi_{A_\pi}(m), \hat{\psi}_{A_\pi}(m) >: m \in H\}",$$

in which the role

$$\psi_{A_\pi}(m) = 2\pi\psi_A(m) \text{ and } \hat{\psi}_{A_\pi}(m) = 2\pi\hat{\psi}_A(m)$$

indicate the degree to which an element m of H belongs to H and meet the criteria $0 \leq \psi_{A_\pi}(m) + \hat{\psi}_{A_\pi}(m) \leq 2\pi$.

Definition 4:"π-Intuitionistic Fuzzy Subgroup (π-IFSG) of a Group"

Let H be a group and π be a set of prime divisors of the order of H. "An A π-IFS A_π of group H is called a π-intuitionistic fuzzy subgroup (π-IFSG) of H if and only if it satisfies the following conditions for all elements m, n ∈ H":

1. Identity: The identity element e of H belongs to A_π, i.e., e ∈ A_π.
2. Closure: For any two elements m, n ∈ A_π, their group product mn also belongs to A_π, i.e., if m, n ∈ A_π, then mn∈ A_π.
3. Inverse: For every element m ∈ A_π, its inverse m^(-1) also belongs to A_π, i.e., if m ∈ A_π, then m^(-1) ∈ A_π.
4. Intuitionistic Fuzzy Set: A_π is an intuitionistic fuzzy set, which means that for each element m ∈ H, there exist membership values μ(m) and non-membership values ν(m) in the closed interval [0, 1], such that:
    - μ(m) reflects the degree to which m is an associate of A_π.
    - ν(m) shows the level of m non-membership in A_π.
    - For each m ∈ H, the complement of μ(m) is given by 1 - μ(m).
    - For each m ∈ H, the complement of ν(m) is given by 1 - ν(m).
    - For each m ∈ H, the condition μ(m) + ν(m) ≤ 1 holds.
5. π-Intuitionistic Fuzzy Subgroup Condition: For all elements m, n ∈ H, "the π-intuitionistic fuzzy set A_π" satisfies the following condition:
    - If π divides the order of the element mn (i.e., the order of mn has prime divisors in π), then the membership value of mn in A_π must be at least the minimum of the membership values of m and n in A_π. Symbolically, if mn has order with prime divisors in π, then μ(mn) ≥ min(μ(m), μ(n)).

Definition 5: Allow a group homomorphism $f: H \to G$ to map to another group G. For any $x$in the range "H and G let A and B be two CIFSG of the groups H and G, respectively". An image of A is the set $f(A)(m) = "\{(m, f(\theta_A)(m), f(\hat{\theta}_A)(m))\}"$

$$f(\theta_A)(m) = \begin{cases} \sup\{\theta_A(x), & \text{if } f(x) = m\}, f^{-1}(m) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}$$

$$f(\hat{\theta}_A)(m) = \begin{cases} \inf\{\hat{\theta}_A(x), & \text{if } f(x) = m\}, f^{-1}(m) \neq \emptyset \\ 1, & \text{otherwise.} \end{cases}$$

The set $f^{-1}(B)(x) = \left\{\left(x, f^{-1}(\theta_B)(x), f^{-1}(\hat{\theta}_B)(x)\right)\right\}$" is "called pre image of B, where"

$$f^{-1}(\theta_B)(x) = (\theta_B)(f(x))$$

$$f^{-1}(\hat{\theta}_B)(x) = (\hat{\theta}_B)(f(x)), \quad \forall x \in H$$

Theorem:"Let $f: H \to G$ be homomorphism from group H to group G. Let A be IFSG of H and B be IFSG of G. Then $f(A)$ is IFSG of G and $f^{-1}(B)$ is IFSG of H".

Lemma:"Let $f: H \to G$ be a homomorphism from group H to group G. Let A and B be two CIFSG. Then"

1 "$f(\theta_A)(m) = f(\eta_A)(m)e^{if(\varphi_A)(m)}$", for all $m \in G$,

2 "$f(\hat{\theta}_A)(m) = f(\hat{\eta}_A)(m)e^{if(\hat{\varphi}_A)(m)}$", for all $m \in G$,

3 "$f^{-1}(\theta_B)(x) = f^{-1}(\eta_B)(x)e^{if^{-1}(\varphi_B)(x)}$", for all $x \in H$,

4 "$f^{-1}(\hat{\theta}_B)(x) = f^{-1}(\hat{\eta}_B)(x)e^{if^{-1}(\hat{\varphi}_B)(x)}$", for all $x \in H$.

Proof:

1 Suppose that

$$f(\theta_A)(m)$$
$$" = \max\{\theta_A(x), \text{if } f(x) = m\}"$$
$$" = \max\{\eta_A(x)e^{if(\varphi_A)(x)}, \text{if } f(x) = m\}"$$
$$= \max\{\eta_A(x), \text{if } f(x) = m\}e^{i\max\{(\varphi_A)(x), \text{ if } f(x)=m\}}$$
$$" = f(\eta_A)(m)e^{if(\varphi_A)(m)}."$$

Hence,

$f(\theta_A)(m) = f(\eta_A)(m)e^{if(\varphi_A)(m)}$.

Assume that

$$f(\hat{\theta}_A)(m)$$
$$" = \min\{\hat{\theta}_A(x), \text{if } f(x) = m\}"$$
$$" = \min\{\hat{\eta}_A(x)e^{if(\hat{\varphi}_A)(x)}, \text{if } f(x) = m\}"$$
$$= \min\{\hat{\eta}_A(x), \text{if } f(x) = m\}e^{"i\min\{(\hat{\varphi}_A)(x), \text{ if } f(x)=m\}"}$$
$$" = f(\hat{\eta}_A)(m)e^{if(\hat{\varphi}_A)(m)}."$$

Hence,

"$f(\hat{\theta}_A)(m) = f(\hat{\eta}_A)(m)e^{if(\hat{\varphi}_A)(m)}$".

Consider,

$$\begin{aligned} f^{-1}(\theta_B)(x) &= \theta_B(f(x)) \\ &= \eta_B(f(x))e^{i\varphi_B(f(x))} \\ &= f^{-1}(\eta_B)(x)e^{if^{-1}(\varphi_B)(x)} \end{aligned}$$

Consequently,

$f^{-1}(\theta_B)(x) = f^{-1}(\eta_B)(x)e^{if^{-1}(\varphi_B)(x)}$.

Consider,

$$f^{-1}(\hat{\theta}_B)(x) = \hat{\theta}_B(f(x))$$
$$= \hat{\eta}_B(f(x))e^{i\hat{\varphi}_B(f(x))}$$
$$= f^{-1}(\hat{\eta}_B)(x)e^{if^{-1}(\hat{\varphi}_B)(x)}.$$

Consequently,

$f^{-1}(\hat{\theta}_B)(x) = f^{-1}(\hat{\eta}_B)(x)e^{if^{-1}(\hat{\varphi}_B)(x)}$.

The results below indicate that there is a homomorphic photograph of CIFSG is always CIFSG.

4. Conclusion

The terms -IFSG (-intuitionistic fuzzy subgroups), CIFSG ("complex intuitionistic fuzzy subgroups"), level subsets, and the intersection of CIFS ("complex intuitionistic fuzzy sets") have all been presented and defined in this article. We have proven the claim that each CIFSG produces two IFSGs and explored its most important consequences at length. We have also investigated the algebraic features of level subsets of CIFSGs and shown that they form a subgroup inside the group. The pre-image and homomorphic representation of an object CIFSG are likewise CIFSGs, as we have shown. To further our findings, we have defined the "direct product of CIFS" and checked that it keeps the CIFSG property when applied to the product of two CIFSGs, revealing various crucial discoveries "about this phenomenon". In the future, we hope to extend the method we've developed here to other algebraic structures and investigate its potential uses in areas like group theory and ring theory. Our ultimate goal is to increase the breadth of our study and aid in the development of algebraic analysis and its applications.

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