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The Study of Heston Option Pricing Model: A Simulation Approach

Jitendra Singh
Shivaji College, University of Delhi, India, New Delhi-110027
Email: jitu.iitd@gmail.com

Abstract

This study explores the application of the Heston model within the framework of Monte Carlo simulation to analyze and forecast financial options pricing and risk management. Compared to standard models that assume constant volatility, the Heston model, which is renowned for its ability to capture stochastic volatility(SV), offers a more realistic portrayal of financial markets. I utilize Monte Carlo simulation techniques to numerically approximate the price of options and evaluate the model's performance across various market conditions.

My approach involves generating paths for both stock price and volatility of the underlying asset using the Heston stochastic differential equations. I then apply these simulated paths to compute option prices and assess the model's sensitivity to different parameters, including volatility of volatility and long-term volatility mean. We also investigate the impact of initial conditions and market trends on pricing accuracy and risk metrics.

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1. Introduction

The right, but not the duty, to purchase or sell a security under predetermined terms is provided by an option. A computationally efficient method to assess the approximate value of the required options is required for a robust option market. There isn't a special Monte Carlo technique. Stochastic procedures, such as Monte Carlo methods, rely on the use of random numbers and probability statistics to examine various issues. From economics to nuclear physics to traffic flow regulation, we can apply Monte Carlo methods. In the field of finance, Monte Carlo methods are frequently employed to determine a company's worth, assess corporate project investments, or appraise financial derivatives. Instead of creating standard static and deterministic financial models, financial analysts who wish to create stochastic or probabilistic models are the ones who should use the Monte Carlo method. for application in the insurance sector[1].

Empirical observations of volatility reveal that, despite appearances, volatility differs. In a way, one would want a volatility model that relies on few factors and represents unpredictability. The models of stochastic volatility involving Brownian motion appear to be the best option. The Hull-White, Scott, and Heston models are a few well-known examples of stochastic volatility models.The Brownian motions in these models are correlated[4].

2. Monte Carlo Method

The term "Monte Carlo method" refers to a broad and popular class of techniques rather than a specific technique. Nonetheless, these methods typically adhere to a specific pattern [2]:
(1)Specifythe range of potential inputs.
(2) Randomly generate inputs from the domain.
(3) Use the inputs to carry out a deterministic computation.
(4) Combine the outcomes of each calculation to arrive at the ultimate outcome.
(5) Performed as often as preferred.

3. The computation of Option Prices using Monte Carlo Simultion

The Monte Carlo simulation can now be used to calculate option pricing. Assume that geometric Brownian motion is followed by the stock price S_t[3]

$$S_t = S_0 exp\left\{\left(r - \frac{1}{2}\sigma^2\right) t + \sigma dB_t\right\}$$

Where r is interest rate of stock price, σ is volatility in stock price, and B_t is Brownian motion. We know that $B_t \sim \sqrt{t}Z$ Where Z ~Normal(0,1). Therefore, we can write stock price at expire time T as,

$$S_T = S_0 exp\left\{\left(r - \frac{1}{2}\sigma^2\right) T + \sigma\sqrt{T}Z\right\}$$

Suppose K is strike price, Then we can compute approximate value of call option $exp(-r * T) * Max[S_T - K, 0]$ or put option $exp(-r * T) * Max[K - S_T, 0]$ via monte carlo simulation method using following algorithm:
For i=1 to N Compute an Normal(0, 1) sample E_i
Set

$$S_i = S_0 exp\left\{\left(r - \frac{1}{2}\sigma^2\right) T + \sigma\sqrt{T}E_i\right\}$$

Set $V_i = exp(-r * T) * Max[S_T - K, 0]$
End
Set

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} V_i$$

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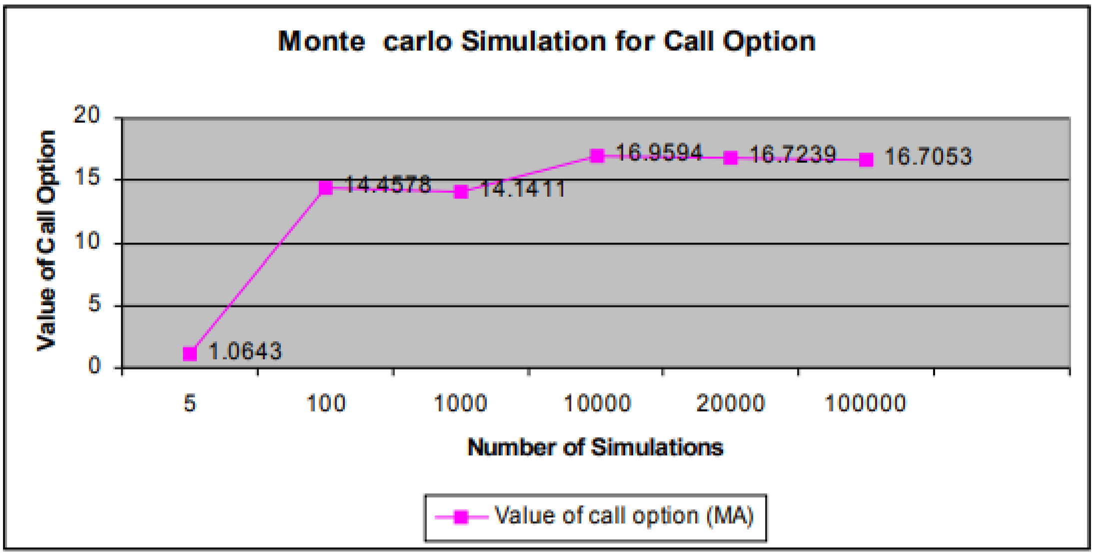
This algorithm will give estimate value of call option

For the following parameters
Time of maturity (T)=4 Years;
Risk-Free-Interest(r) =0.05;
Volatility(σ)=0.2;
Strike Price(K)= $120,
Initial stock Price (S_0)= $100



No of simulation (N)	5	100	1000	10000	20000	100000
Value of call option (MA)	**1.0643**	**14.4578**	**14.1411**	**16.9594**	**16.7239**	**16.7053**

4. The Heston Model

The Heston model is the one of the most popular SV models available today. The strong contrast between its robustness and tractability in comparison to other SV models is what makes it so appealing.Numerous empirical investigations have demonstrated that the log-return of an underlying asset, like stocks, is not necessarily distributed normally. The return and volatility have a negative correlation at the same time. The Black-Scholes model is unable to accurately represent those facts.

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The Heston model, on the other hand, is far more suitable because it can display a wide variety of distributions. The parameter "ρ" is the cause for the several distributions that are presented. It is the correlation between the two dependent Brownian movements, which also illustrates the connection between the underlying asset's volatility and return.
The Heston model's stochastic differential equations (SDEs) are given as follows[4]:

$$dS_t = \mu S_t dt + \sqrt{v_t} S dB_s \ldots\ldots\ldots (i)$$
$$dv_t = k(\theta - v_t)dt + \sigma\sqrt{v_t} dB_v \ldots\ldots\ldots (ii)$$
$$dB_s \circ dB_v = \rho dt$$

where B_S and B_v are two Brownian movements with correlation ρ, and S_t and v_t, respectively, indicate the price and volatility of the underlying asset. A mean reversion procedure is used in the volatility process. The volatility of volatility is denoted by σ, the long-term mean by θ, and the rate of reversion by κ. The relationship between the volatility and the underlying asset is also represented by the correlation. When κ and θ are adjusted to positive values, the volatility drift will diminish as the volatility rises. This characteristic ensures that the volatility doesn't rise unchecked.

5. Simulation of The Heston Model

In order to do our numerical simulations of the Heston Model, we require a discretized version of the continuous-time processes given in Eqs. *(i)* and *(ii)*. For each sufficiently small temporal variation Δt, we employ the standard approximation $B_{t+\Delta t} - B_t \sim \varepsilon\sqrt{t}$, where ε is a random variable and the jumps are generated by the law N(0, 1). The above law of motion can be roughly represented by using Brownian motion's property of the independence of time increments, both for the prices S_t and the instantaneous volatilityv_t as follows

$$S_{t+\Delta t} = S_t + \mu S_t dt + S_t \varepsilon_1 \sqrt{v_t \Delta t} \ldots\ldots\ldots (iii)$$

$$v_{t+\Delta t} = v_t + k(\theta - v_t)\Delta t + \sigma\varepsilon_2\sqrt{v_t \Delta t} \ldots\ldots\ldots (iv)$$

for every time's sequence 0 <t_0< ... <t_n.

For the following parameter, we get fig1 and fig 2 for the Heston Model. Fig 1 show how stock price behave as time increases and Fig 2 show how volatility behave as time changes.

Initial asset price=10;
Initial Volatility=.01;

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Interest rate= .05;

Intensity= 2;

Time=10 years;

number of simulations=100;

Correlation coefficient=-0.9;

Mean of Volatility=.1;

Volatility of Volatility=.01

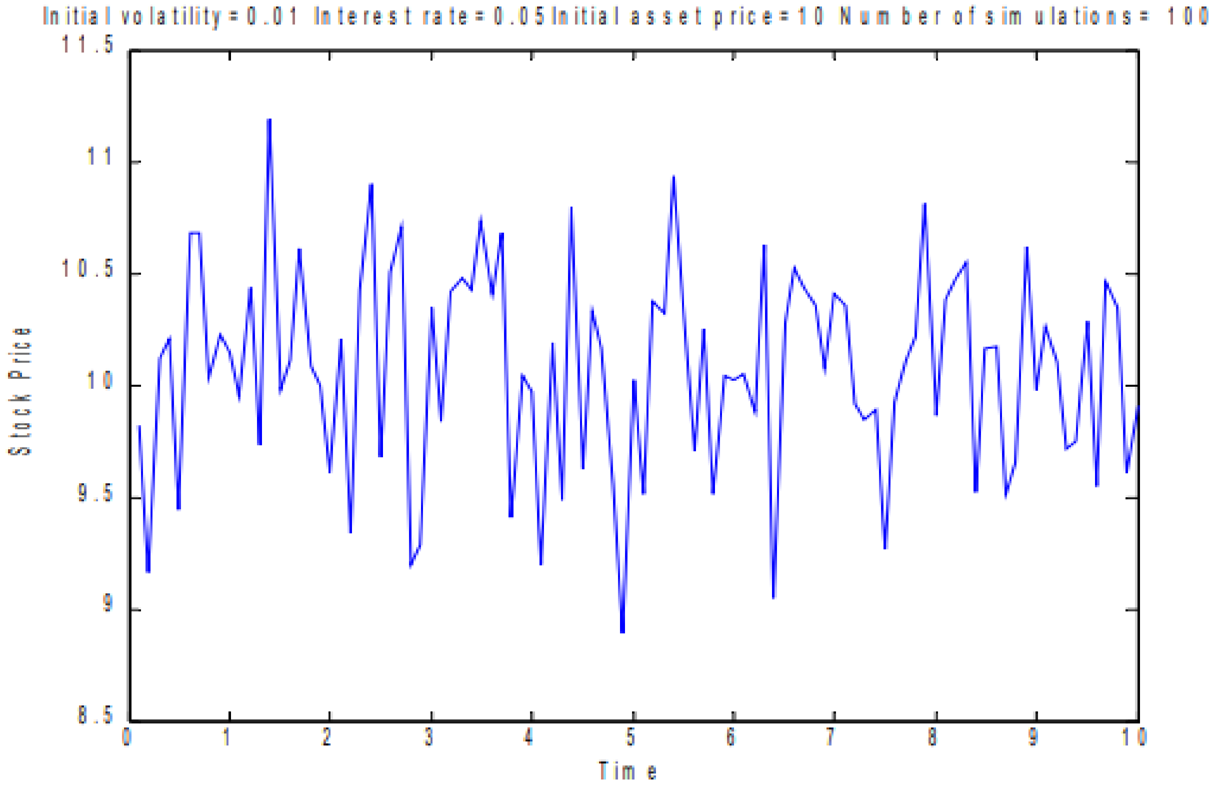

Fig 1: Stock Price Vs Time

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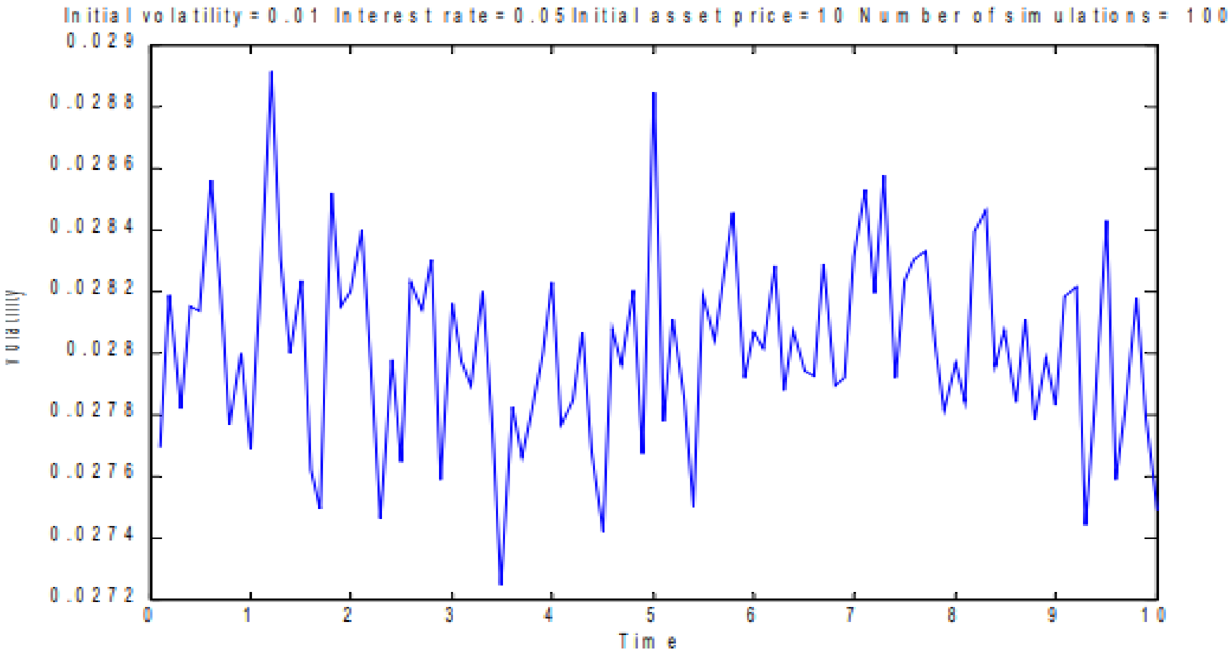

Fig 2: Volatility Vs Time

6. Numerical Illustration

For the following parameters, we get value of call option for the Heston model using monte Carlo Simulation

Initial asset price =100,
Initial Volatility=0.01;
Interest rate= 0.05;
Intensity= 2;
Time=10;
Correlation coefficient= - 0.9;
Strike Price= 90;
Mean of Volatility=.1;
Volatilityof Volatility=.01

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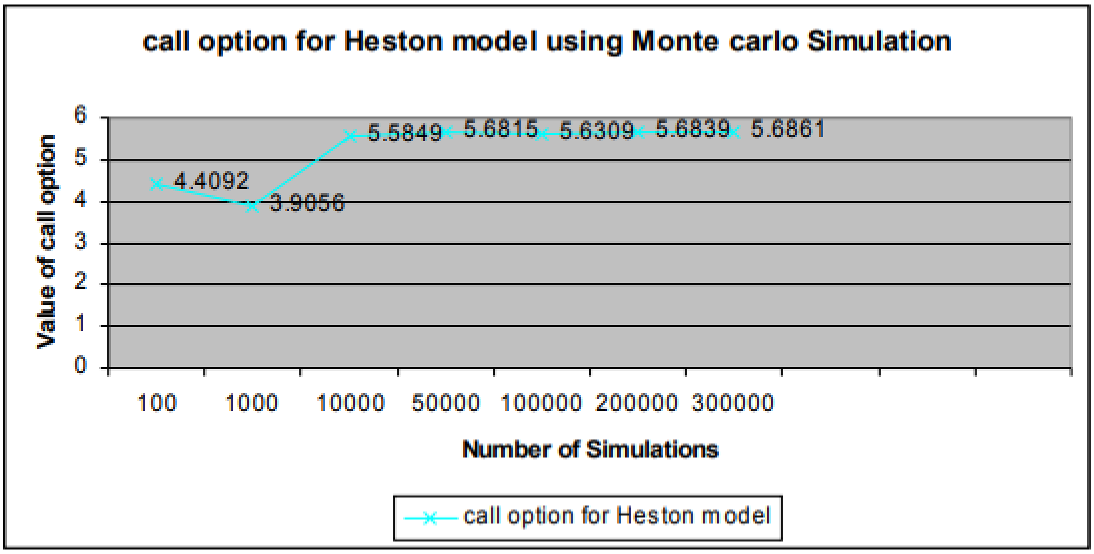

No of simulation (N)	100	1000	10000	50000	100000	200000
Value of call option (MA)	4.4092	3.9056	5.5849	6.6815	5.6309	5.6861

Conclusion:

In this study, we examined the Heston option pricing model using Monte Carlo simulation as a means to value options under stochastic volatility. The Heston model, distinguished by its incorporation of a stochastic process for volatility, provides a more nuanced approach compared to models assuming constant volatility, capturing the empirical observation that volatility itself can vary over time.The Heston model's ability to account for stochastic volatility resulted in more accurate option pricing compared to simpler models like the Black-Scholes model, especially for options with longer maturities or higher volatility. The Monte Carlo simulation allowed us to capture the complex dynamics of the model, which are not easily represented through closed-form solutions. This study underscores the effectiveness of using Monte Carlo simulation to implement the Heston option pricing model. It highlights the model's strength in accommodating stochastic volatility and the simulation's role in accurately valuing options. While computationally intensive, the insights gained from the simulation offer significant advantages in understanding and pricing financial derivatives.

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