
Quantization Characteristics of sigma delta modulation

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ABSTRACT:-

The process of converting an analog signal which has infinite resolution into a finite range number system introduces an error signal that depends on how the signal is being approximated. This quantization error is on the order of one least-significant-bit (LSB) in amplitude, and it is quite small compared to full-amplitude signals. However, as the input signal gets smaller, the quantization error becomes a larger portion of the total signal.

Key Word :- Sigma Delta Modulation, Analog Signal, Analog to Digital Converter, Digital to Analog Converter.

Introduction :-

Quantization is a mathematical term. It is also used in digital signal processing. It is the process of mapping a large set of input values to a countable smaller set. Rounding and truncation are typical examples of quantization processes. Quantization is involved to some degree in nearly all digital signals processing, as the process of representing a signal in digital form ordinarily involves rounding. Quantization also forms the core of essentially all lossy compression algorithms. The difference between an input value and its quantized value, such as round-off error is referred to as quantization error. A device or algorithmic function that performs quantization is called a quantizer. An analog to-digital converter is an example of a quantizer.

When a signal is quantized, the resulting signal approximately has the second-order statistics of a signal with independent additive white noise. Here, we assume that the signal value is in the range of one step of the quantized value. In reality, the quantization noise is of course not independent of the signal; this dependence is the source of idle tones and pattern noise in Sigma-Delta converters. The technique of quantization may be explained with the figure given in figure. Here original signal is represented in the curve of color green. The yellow color curve is quantized signal. The quantization noise s creep with the signal which is shown in brown color curve.

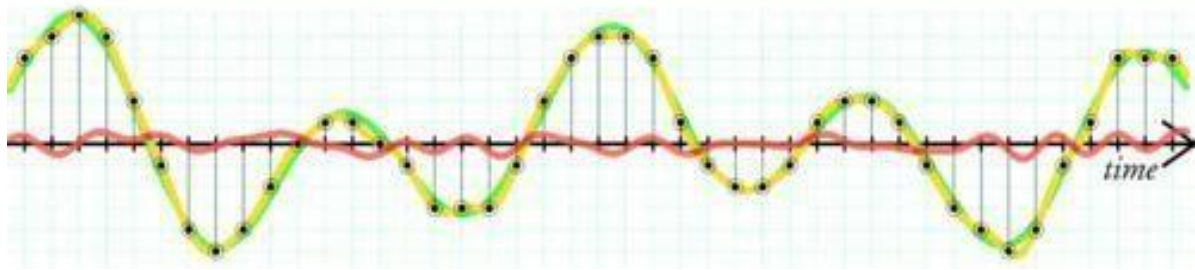


Figure- ORIGINAL SIGNAL, QUANTIZED SIGNAL, QUANTIZATION NOISE

The simplest way to quantize a signal is to choose the

digital amplitude value closest to the original analog (green), the quantized signal (black dots), the signal amplitude. This example shows the original analog signal

quantization error and, in this simple quantization reconstructed difference original signal betweenfrom the quantized signal (yellow) and the and the thereconstructedoriginal signal signal and is thethe scheme, is a deterministic function of the input signal

Basic properties of quantization:-

Because quantization is a many-to-few mapping, it is an inherently non-linear and irreversible process because the same output value is shared by multiple input values, it is impossible in general to recover the exact input value when given only the output value.

The set of possible input values may be infinitely large, and may possibly be continuous and therefore uncountable, such as the set of all real numbers, or all real numbers within some limited range. The set of possible output values may be finite or Countable. The input and output sets involved in quantization can be defined in a rather general way. For example, vector quantization is the application of quantization to multi-dimensional input data.

Analog-to-digital converter:

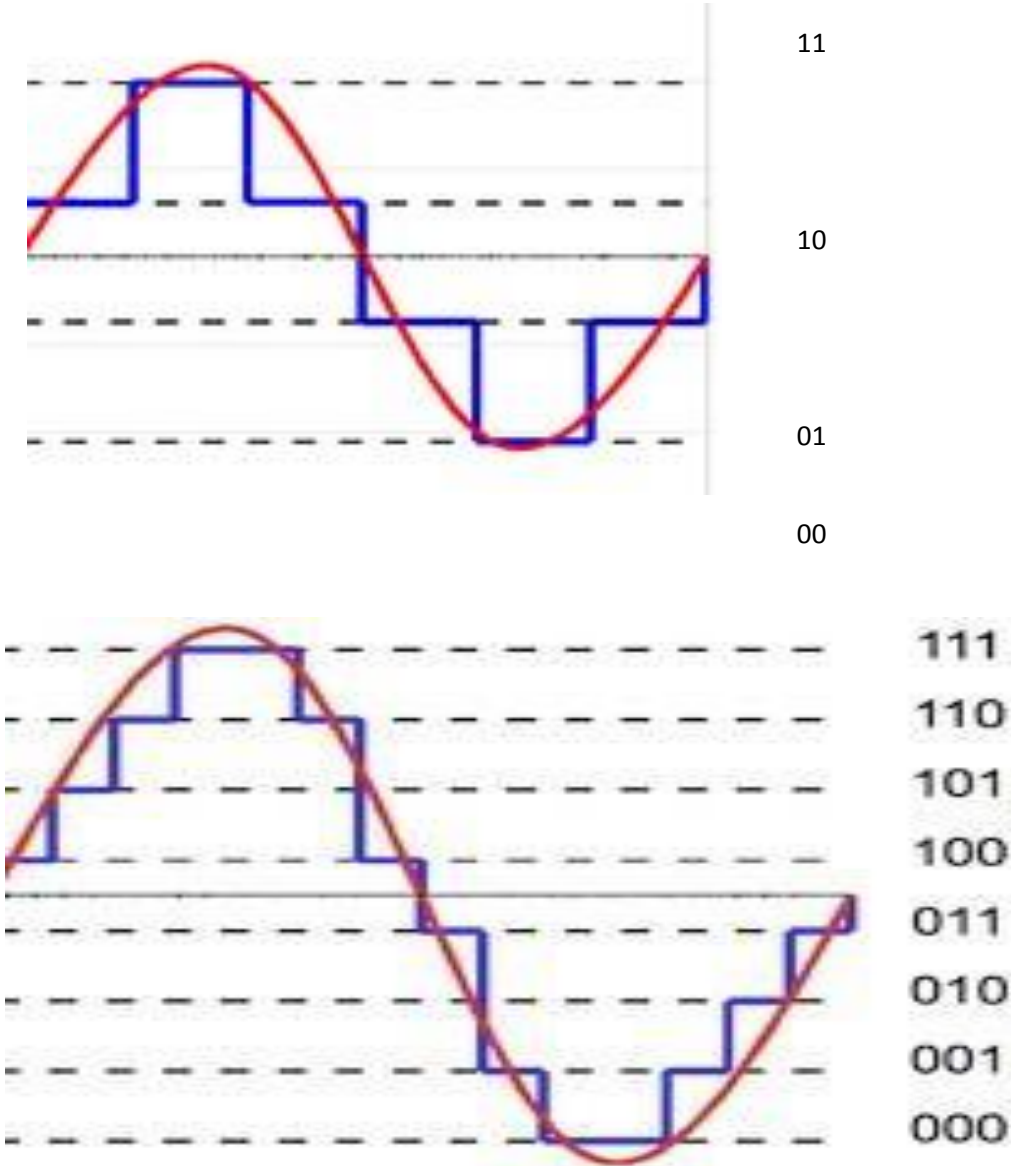
Outside the realm of signal processing, this category may simply be called as rounding or scalar quantization. An ADC processes: sampling and quantization Sampling.

converts a voltage signal function of time into a discrete- time can be modeled as two signals .Quantization replaces each real number with an approximation from a finite set of discrete values, which is necessary for storage and processing by numerical methods. Most commonly, these discrete values are represented as fixed-point words either proportional to the waveform values or floating-

point words. Common word-lengths are 8-bit or 256 levels and 16-bit or 65,536 levels, 32-bit or 4.3 billion levels, and so on, though any number of quantization levels is possible. Quantizing a sequence of numbers produces a sequence of quantization errors which is sometimes modeled as an additive random signal called quantization noise because of its stochastic behavior.

The more levels a quantizer uses, the lower is its quantization noise power. In general, both ADC processes lose some information. So discrete valued signals are only an approximation of the continuous-valued discrete-time signal, which is itself only an approximation of the original continuous-valued continuous-time signal. But both types of approximation errors can be made arbitrarily small by good design.

Figure- 2- bit resolution with four levels of the quantization compared to analog.



Distortion optimization:

Rate–distortion optimized quantization is encountered in source coding for "lossy" data compression algorithms, where the purpose is to manage distortion within the limits of the bit rate supported by a communication channel or storage medium. In this second setting, the amount of introduced distortion may be managed carefully by sophisticated techniques, and introducing some significant amount of distortion may be unavoidable. A quantizer designed for this purpose may be quite different and more elaborate in design than an ordinary rounding operation. It is in this domain that substantial rate– distortion theory analysis is likely to be applied. However, the same concepts actually apply in both use cases.

The analysis of quantization involves studying the amount of data which is generally measured in digits or bits or bit rate, that is used to represent the output of the quantizer, and studying the loss of precision that is introduced by the quantization process (which is referred to as the *distortion*). The general field of such study of rate and distortion is known as rate distortion theory.

Rounding example:

As an example, rounding a real number to the nearest integer value forms a very basic type of quantizer– a uniform one. A typical uniform quantizer with a quantization step size equal to some value can be expressed as,

$$Q(x) = \Delta \cdot \left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor = \Delta \cdot \text{floor} \left(\frac{x}{\Delta} + \frac{1}{2} \right)$$

Where the notation $\lfloor \cdot \rfloor$ or $()$ depicts the floor function. For simple rounding to the nearest integer, the step size is equal to 1. With Δ or with Δ equal to any other integer value, this quantizer has real valued inputs and integer valued outputs, although this property is not a necessity – a quantizer may also have an integer input domain and may also have non-integer output values. The essential property of a quantizer is that it has a countable set of possible output values that has fewer members than the set of possible input values. The members of the set of output values may have integer, rational, or real values (or even other possible values as well, in general, such as vector values or complex numbers).

When the quantization step size is small (relative to the variation in the signal being measured), it is relatively simple to show that the mean squared error produced by such a rounding operation will be approximately $\frac{\Delta^2}{12}$. Mean squared error is also called the quantization noise power. Adding one bit to the quantizer halves the value of Δ , which reduces the noise power by the factor 4. In terms of decibels, the noise power change is: $10 \cdot \log_{10}(\frac{1}{4}) = -6\text{Db}$. Because the set of possible output values of a quantizer is countable, any quantizer can be decomposed into two distinct stages, which can be referred to as the classification stage or forward quantization

stage and the reconstruction stage or inverse quantization stage, where the classification stage maps the input value to an integer quantization index and the reconstruction stage maps the index to the reconstruction value that is the output approximation of the input value. For the example uniform quantizer described above, the forward quantization stage can be expressed as and the reconstruction stage for this example quantizer is simply.

This decomposition is useful for the design and analysis of quantization behavior, and it illustrates how the quantized data can be communicated over a communication channel – a source encoder can perform the forward quantization stage and send the index information through a communication channel possibly applying entropy coding techniques to the quantization indices, and a decoder can perform the reconstruction stage to produce the output approximation of the original input data. In more elaborate quantization designs, both the forward and inverse quantization stages may be substantially more complex. In general, the forward quantization stage may use any function that maps the input data to the integer space of the quantization index data, and the inverse quantization stage can conceptually (or literally) be a table look up operation to map each quantization index to a corresponding reconstruction value. This two stage decomposition applies equally well to vector as well as scalar quantizers.

Uniform quantizers: Most uniform quantizers for signed input data can be classified as being of one of two types: **mid-riser** and **mid-tread**. The terminology is based on what happens in the region around the value 0, and uses the analogy of viewing the input-output function of the quantizer as a stairway. Mid-tread quantizers have a

zero valued reconstruction level (corresponding to a *tread* of a stairway), while mid riser quantizers have a zero valued classification threshold which is corresponding to a *riser* of a stairway. The formulas for mid-tread uniform quantization may be provided in a similar way.

The input-output formula for a mid-riser uniform quantizer is given by:

$$Q(x) = \Delta \cdot \left(\left\lfloor \frac{x}{\Delta} \right\rfloor + \frac{1}{2} \right)$$

Where the classification rule is given by

$$k = \left\lfloor \frac{x}{\Delta} \right\rfloor$$

and the reconstruction rule is

$$y_k = \Delta \cdot \left(k + \frac{1}{2} \right)$$

Note that mid-riser uniform quantizers do not have a zero output value – their minimum output magnitude is half the step size. When the input data can be modeled as a random variable with a probability density function, that is smooth and symmetric around zero, mid-riser quantizers also always produce an output entropy of at least 1 bit per sample.

In contrast, mid-tread quantizers do have a zero output level, and can reach arbitrarily low bit rates per sample for input distributions that are symmetric and taper off at higher magnitudes. For some applications, having a zero output signal representation or supporting low output entropy may be a necessity. In such cases, using a mid-tread uniform quantizer may be appropriate while using a mid-riser one would not be. In general, a mid-riser or mid-tread quantizer may not actually be a uniform quantizer—i.e., the size of the quantizer's classification intervals may not all be the same, or the spacing between its possible output values may not all be the same. The distinguishing characteristic of a mid-riser quantizer is that it has a classification threshold value that is exactly zero, and the distinguishing characteristic of a mid-tread quantizer is that it has a reconstruction value that is exactly zero.

Conclusion :-

In the typical case, the original signal is much larger than one least significant bit (LSB). When this is the case, the quantization error is not significantly correlated with the signal, and has an approximately uniform distribution. In the rounding case, the quantization error has a mean of zero and the RMS value is the standard deviation of this distribution. It may be found that the standard deviation, as a percentage of the full signal range, changes by a factor of 2 for each 1bit change in the number of quantizer bits. The potential signal-to-quantization-noise power ratio therefore changes by 4, or decibels per bit. At lower amplitudes the quantization error becomes dependent on the input signal, resulting in distortion. This distortion is created after the anti-aliasing filter, and if these distortions are above 1/2 the sample rate they will alias back into the band of interest. In order to make the quantization error independent of the input signal, noise with amplitude of 2 least significant bits is added to the signal. This slightly reduces signal to noise ratio, but, ideally, completely eliminates the distortion. It is known as dither.



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