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The onset of Bioconvection in a fluid layer of finite depth subject to an adverse temperature gradient in modulated and unmodulated environments.

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Abstract: Bioconvection is a new area of fluid mechanics and it describes the phenomenon of spontaneous pattern formation in suspension of micro-organisms like Bacillus subtilis and algae. The term "bioconvection" is referred to as macroscopic convection induced in water by the collective motion of a large number of self propelled motile micro-organisms. The direction of swimming micro-organisms is dictated by the different tactic nature of the micro-organisms. They may be gyrotactic, phototactic, chemotactic, etc.. The present study deals with the linear stability analysis of a suspension of gyrotactic micro-organisms in a horizontal fluid layer of finite depth subject to adverse temperature gradient and gravity modulation. This problem is relevant to certain species of thermophilic micro-organisms that live in hot environment.
Keywords: Bio-porous convection, Galerkin Technique, Trial functions,micro-organisms, critical Raleigh number, critical wave number.

1 Introduction: Bioconvection is a new area of fluid mechanics and it describes the phenomenon of spontaneous pattern formation in suspension of micro-organisms like Bacillus subtilis and algae (Pedleys and Kessler 1987,1992). The term "bioconvection" is referred to as macroscopic convection induced in water by the collective motion of a large number of self propelled motile micro-organisms. To the best of author's knowledge the effect of gravity modulation on the interaction of bioconvection and natural convection has not been studied earlier. The direction of swimming micro-organisms is dictated by the different tactic nature of the micro-organisms. They may be gyrotactic, phototactic, chemotactic, etc.. The present study deals with the linear stability analysis of a suspension of gyrotactic micro-organisms in a horizontal fluid layer of finite depth subject to adverse temperature gradient and gravity modulation. By applying modified Galerkin Technique, the analysis is carried out in detail. The computed results are presented through graphs and are in excellent agreement with the available results in the limiting cases. The present study has several geophysical applications. For example, this problem is relevant to certain species of thermophilic micro-organisms that live in hot environment.

2 Mathematical model: Under the following major assumptions (see chapter 2) viz. (i) Heating from below is sufficiently weak so that the micro-organisms will not be killed and they will retain their gyrotactic nature (ii) Due to the small flow velocity associated with the phenomenon, the inertia terms in the Navier-Stoke's equations are neglected and (iii) the system is subject to modulation in the gravity field, the governing equations are:

$$\rho_w \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad ..(1)$$

$$\rho_w \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \quad ..(2)$$

$$\rho_w \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - n\theta\Delta\rho g + \rho_w g\beta(T - T_o) \quad ..(3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad ..(4)$$

$$c_p\rho_w\left(\left(\frac{\partial T}{\partial t}\right) + u\left(\frac{\partial T}{\partial x}\right) + v\left(\frac{\partial T}{\partial y}\right) + w\left(\frac{\partial T}{\partial z}\right)\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad ...(5)$$

$$\frac{\partial n}{\partial t} = -div(j) \quad ..(6)$$

where

$$\vec{J} = n\vec{v} + nW_c\hat{P} - D\nabla n \quad ..(7)$$

Nomenclature : ρ_w : The density of water , $\vec{v} = (u, v, w)$: The fluid convection velocity vector, t : The time, p : The excess pressure above the hydrostatic, $\hat{p}$: The unit vector in the direction of swimming of micro-organism (x,y,z) : Cartesian co-ordinates (space variables), μ : The dynamic viscosity of the suspension, $W_c\hat{P}$: The vector of average swimming velocity relative to the fluid ($W_c =$ constant), β : The volume expansion coefficient of water at constant pressure**,** $\Delta\rho$: $\rho_{cell} - \rho_w$ (the density difference)**,** $\vec{J}$: The flux of micro-organisms due to microscopic convection of the fluid, self propelled swimming of micro-organisms and the diffusion of micro-organisms (further, it is assumed that all random motions of micro-organisms can be approximated by a diffusive process)**,** c_p : The specific heat of water;**,** D : The diffusivity of microorganisms; $\vec{g}$ is acceleration due to gravity**,** k : The conductivity of water**,** n : The number density of motile organisms**,** Q : The bioconvection $Pe'clet$ number $\vec{g} = g_o G^*$ where G^* : modulation parameter, $G^* = 1 + (g(t)/g_o)$

where $g = g(t) + g_o$ is the acceleration due to gravity

Boundary conditions

A horizontal fluid layer of depth H is considered. cartisian axes with the z-axis vertical are utilized, so that the layer is confined between z=0 and z=H. It is assumed that the layer is unbounded in the x and y directions.

At the bottom of the layer (assumed to be rigid), the following conditions are satisfied. At z=0:u=v=w=0, T=$T_o + \Delta T, \vec{J}.\hat{k} = 0$ ..(8) where $\hat{k}$ is the vertically upward unit vector.

The upper surface of the layer is assumed rigid as well because, according to Hill et.al. [6.2], even if it is open to the air, micro-organisms tend to collect at the surface forming what appears to be a packed layer, and it is unlikely that the upper boundary is ever fully stress free. Under this assumption, the boundary conditions at upper surface of the layer are, u=v=w=0, T=T_o, $j.\hat{k} = 0$ at z=H ..(9)

3 Solution procedure(Basic state)

In the basic state the equation of continuity admits a steady state solution where the fluid is motionless and n_b the number density of the micro-organisms in the basic state, p_b the pressure in the basic state and T_b the temperature in the basic state, are functions of z only. In this case equations (6) and (7) r

$$n_b W_c = D\frac{\partial n_b}{\partial z}\cdot$$
..(10)

The solution of this equation is $n_b(z) = \nu \exp\left(\frac{W_c z}{D}\right)$
..(11)

The integration constant ν, which represents the value of the basic number density at the bottom of the layer, is related to the average concentration

$$\bar{n} = \frac{1}{H}\int_0^H n_b(z)dz = \frac{\nu}{H}\int_0^H \exp\left(\frac{W_c z}{D}\right)dz \quad ..(12)$$ and

so is given by,

$$\nu = \frac{\bar{n}Q}{\exp(Q)-1}. \quad ..(13)$$

where the bioconvection Peclet number Q is defined by

$$Q = \frac{W_c H}{D} \quad ..(14)$$

From equations (5), (8), (9), the temperature distribution in the basic state is from (5)

$$T_b = -\frac{\Delta T}{H}Z + T_0 + \Delta T \quad ..(15)$$

From (3) the pressure distribution in the basic state is found from integrating the following equation

$$\frac{\partial p}{\partial z} = -\nu\theta\Delta\rho g \exp\left(\frac{W_c z}{D}\right) + \rho_w g\beta\Delta T\left(1-\frac{z}{H}\right) \quad ..(16)$$

$$p_b - p_0 = \nu\theta\Delta\rho g\frac{D}{w_c}\left[\exp(Q) - \exp(\frac{wcz}{D})\right] - \rho_w g\beta\Delta T \quad ..(17)$$

$$\left[H - z - \frac{1}{2H}\left(H^2 - z^2\right)\right]$$

Linear stability analysis

The perturbations are introduced as follows: n(t,x,y,z)=$n_b(z) + \varepsilon n*(t,x,y,z)$
..(18)

$$v(t,x,y,z) = \varepsilon v*(t,x,y,z)$$
..(19)

$$p(t,x,y,z) = p_b(z) + \varepsilon p*(t,x,y,z) \quad ..(20)$$

$$T(t,x,y,z) = T_b(z) + \varepsilon T*(t,x,y,z) \quad ..(21)$$

$$\hat{p}(t,x,y,z) = \hat{k} + \varepsilon\hat{p}*(t,x,y,z) \quad ..(22)$$

where * denotes a perturbation quantity and ε is the small perturbation amplitude. Substituting equations (6.19)-(6.23) into equations (6.1)-(6.7) and linear zing results in the following equations for perturbation:

$$\rho_w\frac{\partial u*}{\partial t} = -\frac{\partial p^*}{\partial x} + \mu\left(\frac{\partial^2 u*}{\partial x^2} + \frac{\partial^2 u*}{\partial y^2} + \frac{\partial^2 u*}{\partial z^2}\right) \quad ..(23)$$ Similarly,

$$\rho_w\frac{\partial v*}{\partial t} = -\frac{\partial p*}{\partial y} + \mu\left(\frac{\partial^2 v*}{\partial x^2} + \frac{\partial^2 v*}{\partial y^2} + \frac{\partial^2 v*}{\partial z^2}\right) \quad ..(24)$$

$$\rho_w\frac{\partial w^*}{\partial t} = -\frac{\partial p^*}{\partial y} + \mu\left(\frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w^*}{\partial y^2} + \frac{\partial^2 w^*}{\partial z^2}\right) - n^*\theta\Delta\rho g + \rho_w g\beta T^* \quad ..(25)$$

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}\frac{\partial w^*}{\partial z^*}=0 \qquad ..(26)$$

$$C_p\rho_w\left(\frac{\partial T^*}{\partial t}-w^*\frac{\Delta T}{H}\right)=k\left(\frac{\partial^2 T^*}{\partial x^2}+\frac{\partial^2 T^*}{\partial y^2}+\frac{\partial^2 T^*}{\partial z^2}\right) \qquad ..(27)$$

$$\frac{\partial n^*}{\partial t}=-div\left[n_b\left(v^*+W_c\hat{p}^*\right)+n^*W_c\hat{k}-D\nabla n^*\right] \qquad ..(28)$$

The elimination of u*,v*, and p* from eqns. (23)-(26) results in i.e results in

$$\rho_w\frac{\partial}{\partial t}\left(\frac{\partial^2 w^*}{\partial x^2}+\frac{\partial^2 w^*}{\partial y^2}+\frac{\partial^2 w^*}{\partial z^2}\right)=-\theta\Delta\rho g\left(\frac{\partial^2 n^*}{\partial x^2}+\frac{\partial^2 n^*}{\partial y^2}\right)+\rho_w g\beta\left(\frac{\partial^2 T^*}{\partial x^2}+\frac{\partial^2 T^*}{\partial y^2}\right)$$
$$+\mu\left(\frac{\partial^4 w^*}{\partial x^4}+\frac{\partial^4 w^*}{\partial y^4}+\frac{\partial^4 w^*}{\partial z^4}+2\frac{\partial^4 w^*}{\partial x^2\partial y^2}+2\frac{\partial^4 w^*}{\partial x^2\partial z^2}+2\frac{\partial^4 w^*}{\partial y^2\partial z^2}\right) \qquad ..(29)$$

Since it is assumed that temperature variation within the fluid layer does not influence gyrotactic behavior of micro-organisms, according to Pedley et al.[1988],for gyrotactic micro-organisms,

$$\hat{p}^*=B(\eta,-\xi,0) \qquad ..(30) \qquad \text{where}$$

$$\xi=(1-\alpha_0)\frac{\partial w^*}{\partial y}-(1+\alpha_0)\frac{\partial v^*}{\partial z} \qquad (31)$$

$$\eta=-(1-\alpha_0)\frac{\partial w^*}{\partial y}+(1+\alpha_0)\frac{\partial u^*}{\partial z} \qquad ..(32)$$

$$\alpha_0=\frac{a^2-b^2}{a^2+b^2} \qquad ..(33)$$

$$B=\frac{\alpha_\perp\mu}{2h\rho_0 g} \qquad ..(34)$$

where a and b are the semi-major and minor axes of the spheroidal cell, α_0 is a measure of the cell eccentricity, B is the gyrotactic orientation parameter which is introduced by Pedley and Kessler[1987] and which has dimension; $\alpha_\perp$ is dimensionless constant relating viscous torque to the relative angular velocity of the cell ; and h is the displacement of center of mass of the cell from the center of buoyancy.

Thus equation (28) can be written as follows,

$$\frac{\partial n^*}{\partial t}=-div\left[n_b(v^*+w_c\hat{p}^*)+n^*w_c\hat{k}-D\nabla n^*\right]$$
$$\frac{\partial n^*}{\partial t}=-div\left[w_c n_b(B(\eta,-\xi,0)\right]-\frac{\partial n^*}{\partial z}W_c+D\nabla^2 n^*-w_c^*\frac{\partial n_b}{\partial z}$$
$$\frac{\partial n^*}{\partial t}=-w^*\frac{\partial n^*}{\partial z}-w_c\frac{\partial n^*}{\partial z}-w_cBn_b\left(\frac{\partial\eta}{\partial x}-\frac{\partial\xi}{\partial y}\right)+D\nabla^2 n^* \qquad ..(35)$$

With the consideration of (31), (32), eqn. (35) can be recast as:

$$\frac{\partial n^*}{\partial t}=-w^*\frac{\partial n_b}{\partial z}-w_c\frac{\partial n^*}{\partial z}+w_cBn_b[(1-\alpha_0)\left(\frac{\partial^2 w^*}{\partial x^2}+\frac{\partial^2 w^*}{\partial y^2}\right)+$$
$$(1+\alpha_0)\frac{\partial^2 w^*}{\partial z^2}]+\mathrm{D}\,\Delta^2 n^* \qquad ..(36)$$

A normal mode expansion is introduced in the following form;

[w*, n*, T*]=[W(z),N(z), $\theta(z)$] $f(x,y)\exp(\sigma t)$ ..(37)

The function f(x,y) satisfies the following equation;

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=-m^2 f \qquad ..(38)$$

where m is the horizontal wave number (used as separation constant). Substituting eqn. (38) into eqns. (28), (30), (37) and accounting for eqn. (38), the following equations for the amplitudes W, Θ , and N, are obtained:

$$\rho_w \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)(Wfe^{\sigma t})=-\theta\Delta\rho g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(Nfe^{\sigma t})$$

$$+\rho_w g\beta\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\Theta fe^{\sigma t})+\mu\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4}{\partial y^4}+\frac{\partial^4}{\partial z^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+2\frac{\partial^4}{\partial x^2\partial z^2}+2\frac{\partial^4}{\partial y^2\partial z^2}\right)$$

$$(Wfe^{\sigma t})\rho_w\sigma e^{\sigma t}(-Wm^2 f+fW^{11})=\theta\Delta\rho gNe^{\sigma t}m^2 f-\rho_w g\beta\Theta m^2 fe^{\sigma t}+\mu e^{\sigma t}Wm^4 f$$

$$+\mu fe^{\sigma t}W^{1v}+\mu 2(-m^2 f)W^{11}e^{\sigma t}+2\mu We^{\sigma t}f\theta\Delta\rho gm^2 N-(\mu m^4+\rho_w\sigma m^2)W$$

$$+\rho_w g\beta m^2\Theta+(2\mu m^2+\rho_w\sigma)W''-\mu W^{IV}=0$$

$$\cdot-\theta\Delta\rho gm^2 N-(\mu m^4+\rho_{w\iota}\sigma m^2)W+\rho_{w\iota}g\beta m^2\Theta+(2\mu m^2+\rho_{w\iota}\sigma)W^{11}-\mu W^{1v}=0 \qquad ..(39)$$

Further

$$C_p\rho_w\left(\frac{\partial(\Theta e^{\sigma t}f)}{\partial t}-(Wfe^{\sigma t})\frac{\Delta T}{H}\right)=k\left(\Theta e^{\sigma t}(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2})+\frac{\partial^2(\Theta fe^{\sigma t})}{\partial z^2}\right)$$

$$C_p\rho_w\Theta f\sigma e^{\sigma t}-C_p\rho_w Wfe^{\sigma t}\frac{\Delta T}{H}=-k\Theta e^{\sigma t}m^2 f+ke^{\sigma t}\Theta^{11}f$$

implies

$$-c_p\Delta T\rho_w W+H[(km^2+c_p\rho_w\sigma)\Theta-k\Theta'']=0 \qquad ..(40)$$

From eqn. (36)

$$\sigma Nfe^{\sigma t}=-Wfe^{\sigma t}\exp\left(\frac{W_c z}{D}\right)v\frac{W_c}{D}-w_c N^1 fe^{\sigma t}+w_c B(1-\alpha_0)\exp\left(\frac{W_c z}{D}\right)v(-m^2 f)W$$

$$+11(1+\alpha_0)W^{11}fe^{\sigma t}+D[\nabla_1^2+\frac{\partial^2}{\partial z^2}]Nfe^{\sigma t}$$

$$\Rightarrow \sigma Nfe^{\sigma t}=-Wfe^{\sigma t}\exp\left(\frac{W_c z}{D}\right)v\frac{W_c}{D}-w_c N^1 fe^{\sigma t}+w_c B(1-\alpha_0)\exp\left(\frac{W_c z}{D}\right)v(-m^2 f)We^{\sigma t}$$

$$+(1+\alpha_0)W^{11}fe^{\sigma t}+D^2[-m^2 N+N^{11}]fe^{\sigma t}$$

$$\Rightarrow D\sigma N+W\exp\left(\frac{W_c z}{D}\right)vW_c+Dw_c N^1+w_c B\nu\exp\left(\frac{W_c z}{D}\right)m^2 D(1-\alpha_0)-Dw_c B\vartheta$$

$$\exp\left(\frac{W_c z}{D}\right)(1+\alpha_0)W^{11}+D^2 m^2 N-DN^{11}=0$$

$$D(Dm^2+\sigma)N+D(W_c N^1-DN^{11})-\exp\left(\frac{W_c z}{D}\right)W_c v\times[-(1+BDm^2(1-\alpha_0)) \qquad ..(41)$$

$$W+BD(1+\alpha_0)W'']=0$$

Introducing the following dimensionless variables,

$$\bar{z}=\frac{z}{H},a=mH,\overline{W}=\frac{v\theta W_c H^2}{D^2}W,\overline{N}=N\theta,\overline{\Theta}=\beta\Theta$$

eqns. (40)-(42) can be recast as:

$$-a^2 RbQ\overline{N}-\left(a^4+a^2\frac{\rho_w H^2}{\mu}\sigma\right)\overline{W}+a^2\frac{RbQ}{\omega}\overline{\Theta}+\left(2a^2+\frac{\rho_w H^2}{\mu}\sigma\right) \qquad ..(42)$$

$$\overline{W}''-\overline{W}^{IV}=0$$

$$-\frac{Ra}{Q}\frac{\omega}{Rb}\overline{W}+\left[\left(a^2+\frac{H^2 c_p\rho_w}{k}\sigma\right)\overline{\Theta}-\overline{\Theta}''\right]=0 \qquad ..(43)$$

$$\left(a^2+\frac{H^2}{D}\sigma\right)\overline{N}+\left(Q\overline{N}'-\overline{N}''\right)-\exp(Q\bar{z})\left[-(1+G(1-\alpha_0)a^2)\overline{W}+G(1+\alpha_0)\overline{W}''\right]=0 \qquad ..(44)$$

where $Rb=\Delta\rho gv\theta H^3/(\mu D)$ is the bioconvection Rayleigh number, $\omega=\Delta\rho/\rho_w$ is the measure of density of micro-organisms, $Ra=g\beta\Delta TH^3\rho^2{}_w c_p/(\mu k)$ is the traditional Rayleigh number associated with natural convection, and $G=BD/H^2$ is the gyrotaxis number. For the solution of this system a simple Galerkin procedure is employed as follows.

Galerkin procedure : We consider the boundary conditions and the requirement set of functions which is symmetric about $\bar{z}$ =0. It can be represented as a power series in $\bar{z}$ as

$$\bar{W}_1(\bar{z},t)=\sum_{i=1}^{N} a_i(t)W_i(\bar{z})$$

$$\Theta_1(\bar{z},t)=\sum_{i=1}^{N} b_i(t)\Theta_i(\bar{z})$$

$$\bar{N}_1(\bar{z},t)=\sum_{i=1}^{N} n_i*(t)N_i(\bar{z})$$

The trial functions are,

$$\left.\begin{aligned}\bar{W}_1 &= \bar{z}^2-2\bar{z}^3+\bar{z}^4\\ \bar{\Theta}_1 &= \bar{z}-\bar{z}^2\\ \bar{N}_1 &= 2-Q(1-2\bar{z})-Q^2(\bar{z}-\bar{z}^2)\end{aligned}\right\} \quad ..(48)$$

The important determinant that governs the stability of the bioconvective system is

$$\begin{vmatrix} f_2 & f_3 & f_4\\ f_6 & f_7 & 0\\ f_9 & 0 & f_{10}\end{vmatrix} \quad ..(49)$$ where

$f_2=\left[a^4\langle \bar{w}_i\bar{w}_j\rangle-2a^2\langle \bar{w}_i'' w_j\rangle+\langle \bar{w}_i^{IV}\bar{w}_j\rangle\right]=0.0015873a^4-2a^2(-0.019048)+0.7992$

$f_3=-a^2\dfrac{RbQ}{\omega}\langle\bar{\Theta}_i\bar{w}_j\rangle=a^2(0.007143)$, $f_4=a^2RbQ\langle\bar{N}_i\bar{w}_j\rangle=a^2RbQ(0.0665874)$, $f_6=\dfrac{Ra\omega}{QRb}\langle\bar{w}_i\,\bar{\Theta}_j\rangle=Ra(0.007143)$,

$f_7=-a^2\langle\bar{\Theta}_i\,\bar{\Theta}_j\rangle+\langle\bar{\Theta}_i''\bar{\Theta}_j\rangle=-a^2(0.0333)-0.3333$, $f_9=G(1+\alpha_0)\langle e^{Q\bar{z}}\bar{w}_i''\bar{N}_j\rangle-(1+G(1-\alpha_0))a^2\langle e^{Q\bar{z}}\bar{w}_i\,\bar{N}_j\rangle=$
$10(1.2)(-0.00187219)-9a^2(0.069882949)$ $f_{10}=-a^2\langle\bar{N}_i\,\bar{N}_j\rangle-Q\langle\bar{N}_i'\bar{N}_j\rangle+\langle\bar{N}_i''\bar{N}_j\rangle$
$=-a^2(3.9966453)-Q(0.399948)+0.039967$

By using the recurrence relation

$$\partial(m,n)=\int_0^1 \bar{z}^{m+2}(1-\bar{z})^n\,d\bar{z} \quad .(50)$$

$$\partial_{(m^*,n)}=\frac{n}{m*+n+1}\partial_{(m^*,n-1)}..Where..m^*=m+2 \quad ..(51)$$

Further

$$\left.\begin{aligned}&W_i=\bar{z}^{i+1}(1-\bar{z})^{i+1}.,\\ &.\Theta_i=\bar{z}^{i+1}(1-\bar{z})^{i+1}\\ &and.\\ &N_i=(2-Q)+Q(2-Q)\bar{z}^i+Q^2\bar{z}^{i+1}\end{aligned}\right\} \quad ..(51)$$

$$\langle W_iW_j\rangle=\int[\bar{z}^{i+1}(1-\bar{z})^{i+1}][\bar{z}^{j+1}(1-\bar{z})^{j+1}]d\bar{z}. \quad ..(53)$$ and etc

Using the recurrence relation table (50) for $\partial_{(m^*,n)}$ is constructed

Table : computed values of $\partial_{(m^*,n)}$

	n=0	n=1	n=2	n=3	n=4	n=5	n=6
$.m^*=0$	1	0.5	0.3333	0.25	0.2	0.1666	0.142857
$.m^*=1$	0.5	0.1666	0.0833	0.05	0.03336	0.02381	0.01785
$.m^*=2$	0.3333	0.0833	0.0333	0.01666	0.009428	0.005952	0.003968
$.m^*=3$	0.25	0.05	0.01666	0.007143	0.003571	0.001984	0.001190476
$.m^*=4$	0.2	0.0333	0.009524	0.003571	0.0015873	0.00079365	0.0004329
$.m^*=5$	0.1667	0.02381	0.005952	0.001984	0.00079365	0.00036075	0.00018038

Finaly $Rb_{crit}=$ 15.01785623*((.15e-2*a^4+.38096e-1*a^2+.79+.15e-2*a^2*P+.192e-1*P)*(-.3333e-1*a^2-.3333)*(-a^2*((2-Q)^2+1.0*Q*(2-Q)^2+.3333*Q^2*(2-Q)*(4-Q)+.50*Q^3*(2-Q)+.2*Q^4)-Q*(Q*(2-Q)^2+1.0*Q^2*(2-Q)+.5*Q^2*(2-Q)^2+.9999*Q^3*(2-Q)+.50*Q^4)+2*Q^2*(2-Q)+1.0*Q^3*(2-Q)+.6666*Q^4)+.51408e-4*a^2*Ra*(-a^2*((2-Q)^2+1.0*Q*(2-Q)^2+.3333*Q^2*(2-Q)*(4-Q)+.50*Q^3*(2-Q)+.2*Q^4)-Q*(Q*(2-Q)^2+1.0*Q^2*(2-Q)+.5*Q^2*(2-Q)^2+.9999*Q^3*(2-Q)+.50*Q^4)+2*Q^2*(2-Q)+1.0*Q^3*(2-Q)+.6666*Q^4))/(Q*a^2*(-.3333e-1*a^2-.3333)*(G*(1+A)*(.8e-3-.4e-3*Q+.4e-3*Q*(2-Q)-.30008*Q^2+.666e-1*Q^2*(2-Q)-.214152*Q^3)-(1+G*(1-A))*a^2*(.666e-1-.333e-1*Q+.3332e-1*Q*(2-Q)+.9524e-2*Q^2*(2-Q)+.9524e-2*Q^2+.5953e-2*Q^3))))

We compute all the terms. Extensive mathematical calculations and computations are involved. Finally the **Critical BioRayleigh** number is computed for suitable values of the parameters and the results are presented through graphs (1) and (2).

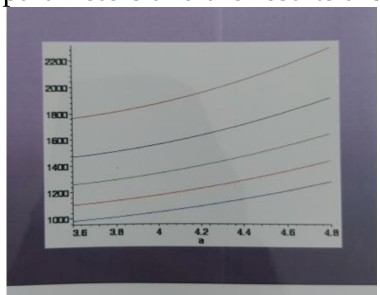

Figure 6.2 : R_a vs a and G*=1, 1.2, 1.4, 1.6 and 1.8

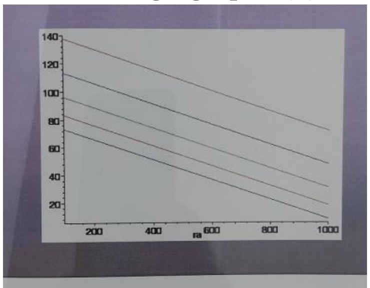

Figure 6.1 : Rb_{crit} vs R_a and G*=1, 1.2, 1.4, 1.6 and 1.8

Results and discussion: The computed results are plotted in figures 1 and 2. In figure 1, the graph of Rb_{crit} *vs* R_a is plotted for Q=0.1, a=4.3 and G^*=1. 12, 1.4 , 1.6 and 1.8. The graph predicts that the Bio-Rayleigh number for a particular value of the modulation parameter. Bioconvection is predominated for moderate values of R_a . Further, as G^* increases, the bio-Rayleigh number decreases for particular values of R_a . Accordingly, $Rb_{crit}|_{G^*=1} \rangle Rb_{crit}|_{G^*\rangle 1}$. In figure 2, the graph R_a *vs* a is plotted for Q=0.1, and G^*=1.2, 1.4 and 1.6 respectively. The figure reveals that Ra increases with a for a particular value of G^* and. $Rb_{crit}|_{G^*=1} \rangle Rb_{crit}|_{G^*\rangle 1}$

Finally, it is concluded that, modulation has a strong influence on bio-convection.

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