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STUDY ON THE IMPLEMENTATION OF SOFT METRIC SPACE IN MATHEMATICS

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ABSTRACT

There are a great number of academics from a wide variety of disciplines that are interested in soft set theory. Solid theoretical breakthroughs are inextricably linked to the successful application of theoretical concepts to the resolution of real-world problems. However, as a result of some attempts to generalize difficult concepts to more lenient situations, structures that are essentially equivalent are produced. Exactly this kind of circumstance is discussed in this essay. The demonstration that a metrizable soft topology may be induced by a soft metric is the primary focus of the work that is being done. The fact that a classical topology is homeomorphic to the soft topology that is provided by a soft metric is the subject of widespread knowledge. Within the scope of this study, the metrizability of this classical topology is shown. An further point to consider is that the classical topology is explicitly imposed by an ordinary metric that is provided. On the other hand, it has been shown that the measurements that are often referred to as soft measurements are, in reality, cone measurements. It has been proved beyond a reasonable doubt that cone metrics are not an effective method for generalizing measurements. Based on these discoveries, it is evident that the classical theory that is linked with soft metric spaces has the potential to be directly used in order to import the great majority, if not all, of the characteristics of these spaces. Finally, the author uses the homemorphism that was produced as a consequence of the findings to demonstrate a new soft fixed point theorem. This homemorphism is an application of the results.

Keywords: - *soft metric space, soft topology, topology, fixed point.*

INTRODUCTION

In order to address the difficulty of dealing with uncertain and imprecise data, a number of different kinds of sets have arisen as potential solutions. As one of these alternatives, Molodtsov put out the idea of soft sets in the year 1999. Through the past several years, there has been a rapid expansion of both the theoretical and practical applications of soft set theory. The introduction of generalizations of soft sets occurred not too much longer after that. Neutronosophic soft sets, N-soft sets, fuzzy soft sets, and soft rough sets are a few examples of these types of sets. When it comes to the challenges of decision-making, Maji et al. is the first of numerous articles that pertain to the topic. These papers include the most recent examples. An algebraic synthesis that made use of soft sets was followed by a number of papers. It was recently demonstrated in that crisp algebras and soft algebras are connected to one another. The concept of soft topology, which is an interface between topology and soft sets, was introduced. Because of this, a significant amount of study has been published in the academic literature since that time. This research includes additional extensions of soft topologies (for instance, readers can go to for more current studies). As a result of the publishing of Matejdes's work, the investigation into soft topology, which is characterized by the utilization of a particular concept of soft points, came to an end. It has been demonstrated by Matejdes that soft topological spaces do, in fact, inherit a topology in the traditional sense. In the end, the study conducted by Alcantud demonstrates without a reasonable doubt that regular topologies and soft topologies are capable of switching places without any noticeable disruption.

With the introduction of the idea of soft metrics, a new area of research was presented. Following the definition of soft metrics, the authors provided evidence that all soft measurements result in the production of a soft topology. In the years that followed, a number of studies were written that discussed fixed-point theorems in relation to this unusual environment. There are a few instances that have been created in recent times. On the other hand, various other efforts have also been made within the realm of generalizing metric spaces. In here, you could discover a survey. The cone metrics did not make it beyond the articles that addressed the metrizability of cone metric spaces due to the fact that they were not included in any of these extensions. Cone metrics do not form a novel class of metric spaces, was the conclusion that was reached by both articles, despite the fact that they used different methods to get at their findings.

It cannot be denied that there has been a large increase in the number of people interested in soft sets, and that a number of different areas have achieved major theoretical and practical advancements in this area. Having said that, we have also seen that when attempting to generalize sharp ideas into similar soft settings, some theories end up being completely identical to one another. To put it another way, the acute and gentle settings possess the capability to be adjusted in both directions. The topic of discussion in this article is a particular case of things of this sort. A crisp metric may be used to quantify the soft topology that is created by a soft metric, which is the major objective of this study. This work aims to demonstrate that this is possible. In order to do this, we will implement a metric that, when applied, will produce a topology that is homeomorphic to the soft topology. This will be accomplished by utilizing the soft metric that has been supplied to us. Moreover, it will be demonstrated that every single soft measure is included in the cone metrics. On the basis of these observations, we are able to confidently determine the topological structure of soft metric spaces. Furthermore, as stated in, the majority of fixed-point theorems have the potential to be converted into the soft setting if the connection between soft metrics, classical metrics, and earlier expansions of metrics is established.

OBJECTIVE

1. Soft metric spaces introduce parameters and soft sets, providing a more robust mathematical structure for dealing with these issues.
2. Soft metric spaces generalize the concept of classical metric spaces by incorporating elements of soft set theory.

Preliminaries

Definition 1. It is not necessary for the sets X and E to empty. denote with the symbol P(X) the collection of all the potential values of X. A soft set over X is the collection of all the potential mappings from E to P(X) that are represented by the pair indicated by the letters (F, E).

Taking into consideration this explanation, the set E represents the parameters that are being considered. The concept of a soft set may be conceptualized in a number of different ways. One way is as a collection of subsets of X that are all connected to one or more of the parameters.

The notation s(E) will be utilized in this investigation to denote the number of items that are contained inside the parameter set E. This is based on the assumption that E is both finite and nonempty. As a result of the fact that the implementation of soft sets is founded on practical concerns rather than the anticipation of an infinite parameter set, this assumption must not to be seen as restrictive. Additionally, several assertions that have been made in the literature are rendered worthless due to the problem of a limitless number of factors.

In the context of X, a soft element is defined as the particular occurrence in which the function $\epsilon: E \rightarrow X$ is defined into X rather than P(X). There is a clear and unmistakable one-to-one relationship between any soft element and any set (A, E) where A(e) is a one-point set. This connection occurs for every e that belongs to the set E.

$(A, E) = \epsilon \Leftrightarrow A(e) = \epsilon(e), \forall e \in E.$

Definition 2. In the context of the real numbers R, the notation B(R) refers to both a set of parameters E and the set of all nonempty bounded subsets of R. An example of a soft real set is a mapping $F: E \rightarrow B(R)$, which is described in the following. After its identification with the accompanying soft element, it will be referred to as a soft real number in the particular scenario where the soft real set (F, E) is a one-point set for every e ∈ E. This will be the case following the identification of the soft element.

Notations such as $\widetilde{m}, \widetilde{n}$, and $\widetilde{r}$ For the purpose of representing soft reals, the symbols m, n, and ¯r are adopted. These symbols represent constant soft real numbers. The situation of constant soft real numbers, in which m(e) = m for every e in E, is one in which we may observe this phenomenon. A collection of parameters E with s(E) = k is all that is required to identify a soft real number with a vector in R k. This identification is an easy process.

$$\widetilde{m} \longleftrightarrow (\widetilde{m}(e_1), \widetilde{m}(e_2), \cdots, \widetilde{m}(e_k)).$$

Definition 3. A partial ordering $\lessapprox$ is defined on the set of soft reals as follows:

i. $\widetilde{m} \lessapprox \widetilde{n}$ *if* $\widetilde{m}(e) \leq \widetilde{n}(e)$, *for all* $e \in E$,

ii. $\widetilde{m} \lesssim \widetilde{n}$ *if* $\widetilde{m}(e) < \widetilde{n}(e)$, *for all* $e \in E$, where $\widetilde{m}, \widetilde{n}$ are soft real numbers.

In light of the fact that concepts such as positivity and arithmetic operations on soft real numbers have meanings that are readily apparent, we do not define them.

Definition 4. A soft set (P, E) X is considered a soft point if, in addition to λ ∈ E and x ∈ X, such that P(λ) = {x} and P(μ) = ∅, ∀μ ∈ E\{λ}. In this case (P, E) will be denoted by Pλx. Definition 5. A soft point Pλx is in a soft set (A, E) if

P(λ) = {x} ⊂ A(λ)

and this will be shown with $P_\lambda^x \widetilde{\in}$(F, E).

As a side note, remember that a soft set is just a collection of all the soft points that belong to it, like, (F, E) $= \cup_{P_\lambda^x \widetilde{\in}(F,E)} P_\lambda^x$.

In the following, $\widetilde{X}$, signifies the boundless soft set that is described by F(λ) = X, ∀λ ∈ E.

S P $(\widetilde{X})$ on behalf of the amassement of all soft spots of $\widetilde{X}$. We shall display non-negative soft real numbers using R(E)∗. A non-rigid measure of S P$(\widetilde{X})$ is defined as follows.

Definition 6. A mapping

$$d : SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$$

If d holds true for every element of the soft set X, we say that it is a soft metric on X. $P_\lambda^x, P_\mu^y, P_\nu^z \widetilde{\in} \widetilde{X}$:

(M1) $d(P_\lambda^x, P_\mu^y) = 0$ if and only if $P\lambda^x = P_\mu^y$,

(M2) $d(P_\lambda^x, P_\mu^y) = d(P_\mu^y, P_\lambda^x)$,

(M3) $d(P_\lambda^x, P_\nu^z) \lessapprox d(P_\lambda^x, P_\mu^y) + d(P_\mu^y, P_\nu^z)$.

The soft set $\widetilde{X}$ applying a moderate measure A soft metric space is denoted by d. The space is shown using the triplee $(\widetilde{X}, d, E)$ or shortly with the pair $(\widetilde{X}, d)$ where the parameter set E is well defined.

An ordered set of open balls characterized by

$$B_d((x, \lambda), \widetilde{r}) = \{P_\mu^y \in SP(\widetilde{X}) : d(P_\lambda^x, P_\mu^y) \lesssim \widetilde{r}\}$$

underpins a soft topology that is based on $\widetilde{X}$.

The fact that the soft metric may be defined on any soft space is really of little consequence. $\widetilde{X}$ or on a regular space X. The range, not the domain, is where the concept of a soft measure differs from a traditional metric.

Definition 7. P is a subset of B, and B is a real Banach space. In this case, P is referred to as a cone:

i. i. P is nonempty and has an open complement $P \neq \{0\}$.
ii. $ax + by \in P$ for all $x,y \in P$ and nonnegative real numbers a,b.
iii. $P \cap (-P) = \{0\}$.

First things first: a cone P ⊂ B, a partial order ⪯ is defined in relation to P by, x ⪯ y whenever y − x ∈ P. x ≺ y will stand for x ⪯ y and x , y, while x << y indicates that y − x ∈ intP, where intP denotes the interior of P.

Definition 8. Let X be a nonempty set and P be a cone. Suppose the mapping d: X × X → P satisfies:

(d1) 0 ⪯ d(x,y) for all x,y ∈ X and d(x,y) = 0 ⇔ x = y;

(d2) d is symmetric;

(d3) d(x,y) ⪯ d(x,z) + d(y,z) for all x,y,z ∈ X.

Then, d is called a cone metric on X and (X,d) is called a cone metric space.

The topology of soft metric spaces

In the original study, it was shown that every soft topological space is really homeomorphic to a topological space on (X × E). However, for consistency with the text, we shall use (X × E) instead. Conversely, it has been shown that a soft topology is induced by every soft metric. In order to get a mutually agreeable configuration, everyone $P_{\lambda}^{x}\tilde{\in}$Xe will be found by matching the ordered pairs (x,λ) that belong to the set (X×E). Therefore, it follows that (X×E) imposes a classical topology for any soft metric. This derived classical topology will be shown to be metrizable in the subsequent section. Additionally, we will derive a metric from the fundamental soft metric that is metrizing the classical topology that was previously discussed. Consequently, the isomorphism between all soft metrics and classical metrics will be obvious. Soft metrics will be defined as vector valued metrics, specifically as a subset of cone metrics, to help shed light on their nature.

Let us reiterate at this juncture that, since a soft metric space $(\widetilde{X}, d)$. in addition to its idea of subtleties $P_{\lambda}^{x}\tilde{\in}$Xe we shall freely use the classical concepts of second countability, regularity and metrizability in the space that follows, as it is in a one-to-one correspondence with a classical topology on (X × E). $(\widetilde{X}, d)$.

Proposition 1. A soft metric space $(\widetilde{X}, d)$. is second countable.

The evidence. Induced soft topology often uses open balls as its foundation. $B_d((x,\lambda), \widetilde{r})$. If the soft real number $\widetilde{r}$ belongs to the set of open balls, sayerq, and can only accept rational values. Bd((x,λ), $\widetilde{r_q}$) is a countable basis for the topology.

Proposition 2. A soft metric space $(\widetilde{X}, d)$ is regular.

Proof. Let K ⊂ S P$(\widetilde{X})$ be a nonempty closed set and $P_\lambda^x \in$ S P$(\widetilde{X})$ be a point not in K. Clearly, d $(P_\lambda^x, K) > \bar{0}$ since d $(P_\lambda^x, K) = 0$ Since Pλx is a closure point of K and so is a part of K, would suggest that. Let

$$2\widetilde{r} = d(P_\lambda^x, K).$$

Consider the open ball

$$V = B_d((x, \lambda), \widetilde{r})$$

and the open set

$$U = \cup_{P_\mu^y \in K}\{B_d((y, \mu), \widetilde{r})\}.$$

Because of the triangle inequality, the open sets U and V cannot overlap with one another. In addition, $P_\lambda^x \in$V and K ⊂ U, so we arrive at the result that $(\widetilde{X})$ is regular.

Corollary 1. It is a metrizability theorem for all soft metric spaces, according to Urysohn.

Naturally, one would wonder whether it is possible to explicitly provide the metric that corresponds to the soft topology. Yes, it is well-known. We shall demonstrate another aspect of soft metrics, namely their link with cone metrics, before moving on to that.

The third proposition states that a cone on $R^{s(E)}$ is the set of all nonnegative soft real numbers $R(E)^*$ associated with the set of parameters E.

The evidence. It is obvious that Rs(E) is a Banach space and that R(E)∗ is a nonempty subset of Rs(E). In addition, $\mathbb{R}(E)^* \neq \{\overline{0}\}$.

(i) We will show that $R^{(E)*}$ is closed with the standard topology on $R^{s(E)}$. Let x ∈ $(R(E)^*)^c$,

$$r = \min_{e \in E}\{|\widetilde{x}(e)|\}, \quad r > 0.$$

Thus, we have

$$B(\widetilde{x}, r) \subset (\mathbb{R}(E)^*)^c.$$

Therefore, $(R(E)^*)^c$ is an open set and R(E)* is closed.

Let ex,ey ∈ R(E)∗ and a,b be two nonnegative real numbers. It is obvious that the linear combination $a\widetilde{x} + b\widetilde{y}$ is a nonnegative vector in $R^{s(E)}$. Therefore, $a\widetilde{x} + b\widetilde{y} \in \mathbb{R}(E)^*$. .

Let $\widetilde{x} \in \mathbb{R}(E)^*$. If $\widetilde{x} \neq \overline{0}$ then $-\widetilde{x} \notin \mathbb{R}(E)^*$. If $\widetilde{x} = \overline{0}$ then $-\widetilde{x} = \overline{0} \in \mathbb{R}(E)^*$. So, we observe that

$$\mathbb{R}(E)^* \cap (-\mathbb{R}(E)^*) = \{\overline{0}\}.$$

Proposition 4. The ordering ⪯ induced by the cone R(E)* coincides with the ordering ≲ defined on R(E)*.

Proof.

$$\widetilde{x} \lessapprox \widetilde{y} \Leftrightarrow \forall \lambda \in \{e_1, e_2, \cdots, e_{s(E)}\}, \quad \widetilde{x}(\lambda) \leq \widetilde{y}(\lambda) \Leftrightarrow \widetilde{x}(\lambda) - \widetilde{y}(\lambda) \leq 0 \Leftrightarrow \widetilde{y} - \widetilde{x} \in \mathbb{R}(E)^* \Leftrightarrow \widetilde{x} \preceq \widetilde{y}.$$

Theorem 1. Let X be an initial universal set, E be a finite set of parameters and d be a soft metric on the soft set $\widetilde{X}$. . Identifying the soft real number $d(P_\lambda^x, P^y_\mu)$ with the corresponding vector in $R^{s(E)}$ the function

$$d_c : (X \times E) \times (X \times E) \to \mathbb{R}^{s(E)},$$

defined by

$$d_c((x,\lambda),(y,\mu)) = \left[d(P^x_\lambda, P^y_\mu)\right]$$

is a cone metric.

Proof: The assertion is an outcome of the reasoning in Propositions 3 and 4.

We can apply the conclusions for cone measurements to soft metrics by noticing that every soft metric is also a cone metric. Specifically, we look at the metrizability of cone metric spaces.

Proof of the metrizing metric of a soft metric space is given in the following theorem.

Theorem 2. Let X be an initial universal set, E be a set of parameters and d be a soft metric on the soft set $\widetilde{X}$. The topology on (X × E) induced by the soft metric d is metrizable with

$$d'((x,\lambda),(y,\mu)) = \|d_c((x,\lambda),(y,\mu))\|_\infty .$$

Proof. The function

$$D((x,\lambda),(y,\mu)) = \|d_c((x,\lambda),(y,\mu))\|_\infty$$

is obviously a metric on (X×E). We will show that D and d induce the same topology on (X×E). To do this, we will show that every open ball $B_d((x,\lambda),\widetilde{r})$ in $(X\times E, d)$ is an open set in (X × E, D) and every open ball $B_d((x,\lambda),r)$ in (X × E, D) is an open set in (X × E,d). Consider the open ball $B_d((x,\lambda),\widetilde{r})$

where 0 ≲er. Let

$$(y,\mu) \in B_d((x,\lambda),\widetilde{r})$$

with $(y,\mu) \neq (x,\lambda).$ Set

$$\widetilde{c} = \widetilde{r} - d(P_\lambda^x, P_\mu^y).$$

Clearly, $\overline{0} \lesssim \widetilde{c}$ and

$$B_d((y,\mu),\widetilde{c}) \subset B_d((x,\lambda),\widetilde{r}).$$

Choosing c′ to be the smallest component of c, we observe that

$$B_D((y,\mu),c') \subset B_d((y,\mu),\widetilde{c}) \subset B_d((x,\lambda),\widetilde{r}).$$

And therefore, each point of the set $B_d((x,\lambda),\widetilde{r})$ is an interior point with respect to the metric D.

So $B_d((x,\lambda),\widetilde{r})$ is an open set in (X × E, D). Conversely, let

$$(y,\mu) \in B_D((x,\lambda),r)$$

with $(y,\mu) \neq (x,\lambda)$ and

$$c = r - D((x,\lambda),(y,\mu))$$

and consider the soft real number $\overline{c}$. Since

$$D((x,\lambda),(y,\mu)) = \|d_c((x,\lambda),(y,\mu))\|_\infty$$

we see that

$$B_d((y,\mu),\overline{c}) = B_D((y,\mu),c) \subset B_D((x,\lambda),r).$$

And therefore, the set BD((x,λ),r) is open in the topological space induced by the soft metric. It should be noted that the metric

$$d' = \|d_c((x,\lambda),(y,\mu))\|_\infty$$

used in the foregoing theorem is called the compatible metric in where the authors used this metric to show that some fixed point theorems in soft metric spaces can be obtained from their classical counterparts. Here we have the main result:

Corollary 2. A soft metric d on a space X is isomorphic to the metrice

$$d' = \|d_c((x,\lambda),(y,\mu))\|_\infty$$

on the space (X × E).

Figure 1 illustrates the complete relation.

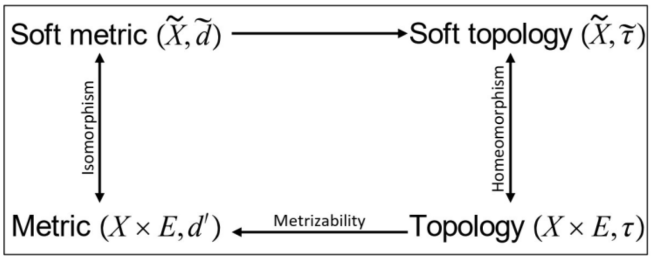

Figure 1. Complete relation between soft and classical metrics.

CONCLUSIONS

In this work, a comprehensive investigation into the field of soft metrics is carried out, and some notable findings are acquired. The soft measurements are proven to include all of the characteristics that are present in the cone metrics. Our findings demonstrate that the space in question is metrizable, and we also provide an isomorphism to a traditional metric space. For the purpose of illustrating the process of transferring fixed point theorems to the soft environment, the findings are used. Additionally, the findings provide insight on the inner workings of soft metrics, which help to a more in-depth understanding of these measures. There is no such thing as a new sort of metrics, and this is true even if the concept is not provided with the intention of establishing a new generalization. Additionally, soft metrics do not provide any innovative findings from a topological point of view. This is due to the fact that they naturally induce a topology that can be accomplished by using a conventional metric.

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