



BAYESIAN ANALYSIS OF TAR FRAMEWORK WITH NUMEROUS STRUCTURAL BREAKS

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Abstract

The estimate and testing techniques may be affected by the impact of a covariate in panel data, which consists of repeated observations across time on the same set of cross-sectional units. In this study, we offer a Bayesian technique to testing and estimating unit roots in a Threshold Autoregressive Model, taking into account structural break. In order to estimate the parameters with a conditional posterior distribution, this article performed a Monte Carlo simulation and compared the results to those obtained with an ordinary least square estimator. Positive findings are obtained from using the estimated posterior odds ratio to test the unit root hypothesis. For greater model utility, we have investigated simulation analyses of fertilizer imports.

Keywords: TAR(Threshold Autoregressive Model) model; Bayesian inference; Gibbs sampler and M-H method approaches

Introduction

Those that devised an optimizer for achieving minimize ideal approach of predictions and created important because in order. Also, a statistical power centered on random effects model was developed and put in place to the expansion of coniferous trees in the China area. This segment discusses that although the inferences about the TAR paradigm with institutional adjustment was made using the classical technique, it was not done using the Naive bayes classifier. From a Bayesian perspective, an unconditional conditional variance has been derived for Bayesian inference. We use a variety of both symmetrical and asymmetrical principal component analysis to construct superior inference. Gibbs estimator and N-H method approaches were applied in a simulated investigation to track how well the Bayesian viewpoint of multi interruptions and different regime TAR model performed (Box et al., 2015).

Proposed Model

Let $\{y_t\}$ be a random variable with numerous unknown variables and intervals at each variable that adhere to several regimes of the TAR model.

$$y_t = \theta^{(ij)} + \sum_{l=1}^{p_{ij}} \phi_l^{(ij)} y_{t-l} + e_t^{(ij)}, \quad r_{ij-1} < y_{t-d_i} < r_{ij}, \quad T_{i-1} < t < T_i \quad (1.1)$$

where, r_{ij} If “j=1,2,...,k i, i=1,2,...,m” and fulfil, does the criterion parameters of the j"th" era of something like the break point create a partitions on the regular grid? The ith delay component of the

TAR type is $\rightarrow i0r i1r (iki)=,d i$. get a significant integer number from 1, 2,..., d i0, where d i0 is the greatest delay lag taken into account. The points where there are structural fractures are $0=T OT 1T m=T$. The numbers of endogeneity (m) that have been divided so that each breaks time follows the TAR model's k i regimens with the d i latency value. The intercept, $e t((ij))$, is an unpredictable random process with an i.i.d. normal distribution and a mean of 0. Moreover, the TAR female's values $_l((ij))$ and lag order $p ij$ vary when breakpoints are present. Repetitive observation in a data set are useful for drawing sensible statistical conclusions. A sequence from the TAR model, therefore, carries the constraint of the paradigm. Hence, MB-TAR

This distinction may be found on an order form without knowing the threshold amount. "Let p be the first findings (y_1, y_2, \dots, y_p) of the series are assumed to be known and π_{ij} be the time index of the j^{th} smallest observation in i^{th} break point in this series. Then, each regime has $(s_{ij} - s_{ij-1})$ observations from $(y_{p+1-d}, y_{p+2-d}, \dots, y_{T_i-d})$, where, $p = \max(p_{ij}; i = 1, 2, \dots, m, j = 1, 2, \dots, k_i)$." the equation (4.76) as

$$y_{\pi_{w_{ij}+d_i}} = \theta^{(ij)} + \sum_{l=1}^{p_{ij}} \phi_l^{(ij)} y_{\pi_{w_{ij}+d_i-l}} + e_{\pi_{w_{ij}+d_i}}^{(ij)}, \quad s_{ij-1} < w_{ij} < s_{ij}, \quad T_{i-1} < s_{ij} < T_i. \dots (4.77)$$

In matrix, the form of the model is

$$Y_{ij} = \Phi^{(ij)} X_{ij} + e^{(ij)} \dots (1.2)$$

where, s_{ij} satisfy $y_{\pi_{e_{ij}}} \leq r_{ij} < y_{\pi_{i_{ij}+1}}, s_{i0} = T_{i-1}, s_{ik_i} = T_i$,

$$n_{ij} = s_{ij} - s_{ij-1},$$

$$Y_{ij} = (y_{\pi_{s_{ij-1}+1+d_i}}, y_{\pi_{s_{ij-1}+2+d_i}}, \dots, y_{\pi_{s_{ij}+d_i}}),$$

$$X_{ij} = \begin{pmatrix} 1 & y_{\pi_{s_{ij-1}+1+d_i-1}} & \dots & y_{\pi_{s_{ij-1}+1+d_i-p_{ij}}} \\ 1 & y_{\pi_{s_{ij-1}+2+d_i-1}} & \dots & y_{\pi_{s_{ij-1}+2+d_i-p_{ij}}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & y_{\pi_{s_{ij}+d_i-1}} & \dots & y_{\pi_{s_{ij}+d_i-p_{ij}}} \end{pmatrix}.$$

Probable function of the system (1.2) is

$$L(\Theta | y) = \prod_{i=1}^m \prod_{j=1}^{k_i} \left((2\pi\sigma_{ij}^2)^{-\frac{n_{ij}}{2}} \exp \left[-\frac{1}{2\sigma_{ij}^2} (Y_{ij} - \Phi^{(ij)} X_{ij})' (Y_{ij} - \Phi^{(ij)} X_{ij}) \right] \right) \dots (1.3)$$

where, $\Theta = \{\Phi^{ij}, \sigma_{ij}^2, r_{ij}, d_i, T_B\}$. In academia, the dependent and independent variables model is often used to address changes in a series' form brought on by unknown quantities, although occasionally change happens with its own previous observations. If it is combined with structural break as a result of variation in data making process with realm of time then, the TAR model is convenient to examine this state gives a productive inference to justify the impact in real application, where series generation contains this phenomenon. For making inference, present chapter considers employing a Bayesian framework to calculate the MB-TAR constraints. With the Bayesian technique, we have extra knowledge about the training sets, results in a sophisticated mathematical

form for the state vector. Parallel incorporation is involved in this form, which may be solved using a variety of Bayesian numerical algorithms

Bayesian Estimation

In contrast to prior knowledge, or current data evidence, Bayesian inference often offers some additional context well about unknown parameter. The choice of a previously is a crucial task since the distinct characteristics of the random variables are effectively explained in relation to a proper prior. The previous convictions of something like the indices from the current research, which simply aims to estimate, have been taken into consideration. The forms of the priors are as listed below,

- (i) Φ^{ij} trailsanself-determining $N(\Phi_0^{ij}, \sigma_{ij}^2 I_{ij})$.
- (ii) σ_{ij}^2 trailsanself-determining inverse gamma distribution $IG(\mu_{ij}, \nu_{ij})$.
- (iii) r_{ij} trailsaunvarying distribution $U(a_{ij}, b_{ij})$.
- (iv) d_i trails a distinct uniform distribution $\{1, 2, \dots, d_{i0}\}$.

With the given number of breaches, we refer to just a process to locate the break positions in a number and believe that all potential locations for rest periods have always had an equal chance of changing the shape of a series, in contrast to many writers like **(Chen et al. 2011)**; **(De Wachter and Tzavalis 2012)** considered unknown number of break and estimated using information criterion. Hence, conditional prior probability of break point location (T_B) under given known number of breaks (m) is defined by Inclan procedure as

$$P(T_B | m) = \frac{1}{\binom{T-1}{m}}$$

The joint prior distribution is

$$\pi(\Theta) = \frac{\prod_{i=1}^m d_{i0}^{-1}}{\binom{T_m - 1}{m}} \prod_{i=1}^m \prod_{j=1}^{k_i} \left(\frac{(2\pi)^{-\frac{p_{ij}+1}{2}} (\sigma_{ij}^2)^{-\frac{p_{ij}+1}{2} + \mu_{ij} + 1} \nu_{ij}^{\mu_{ij}}}{(b_{ij} - a_{ij}) \Gamma(\mu_{ij})} \right) \dots (1.4)$$

$$\exp \left[- \sum_{i=1}^m \sum_{j=1}^{k_i} \frac{1}{2\sigma_{ij}^2} \left\{ (\Phi^{(ij)} - \Phi_0^{(ij)})' I_{ij}^{-1} (\Phi^{(ij)} - \Phi_0^{(ij)}) + 2\nu_{ij} \right\} \right]$$

The random variable provided in (8.4) is then combined with the joint likelihood function in to produce the probability of the current proposal(8.5),

$$\pi(\Theta | Y) = \frac{\prod_{i=1}^m d_{i0}^{-1}}{\binom{T-1}{m}} \prod_{i=1}^m \prod_{j=1}^{k_i} \left(\frac{(2\pi)^{-\frac{n_{ij}+p_{ij}+1}{2}} (\sigma_{ij}^2)^{-\frac{n_{ij}+p_{ij}+1}{2} + \mu_{ij} + 1} v_{ij}^{\mu_{ij}}}{(b_{ij} - a_{ij}) \Gamma(\mu_{ij})} \right) \exp \left[- \sum_{i=1}^m \sum_{j=1}^{k_i} \frac{1}{2\sigma_{ij}^2} \left\{ (Y_{ij} - \Phi^{(ij)} X_{ij})' (Y_{ij} - \Phi^{(ij)} X_{ij}) + (\Phi^{(ij)} - \Phi_0^{(ij)})' I_{ij}^{-1} (\Phi^{(ij)} - \Phi_0^{(ij)}) + 2v_{ij} \right\} \right] \dots (1.5)$$

The probabilistic likelihood function for each component is generated by solving the equations in the Bayesian framework to arrive at the estimate (1.5). The distribtes of conditionally posteriors are derived in the following forms:

$$\Phi^{ij} | Y, \sigma_{ij}^2, r_{ij}, d_i, T_B \sim MN \left(P_{ij} (X'_{ij} X_{ij} + I_{ij}^{-1})^{-1}, \sigma_{ij}^2 (X'_{ij} X_{ij} + I_{ij}^{-1})^{-1} \right) \dots (1.6)$$

where, $P_{ij} = (X'_{ij} Y_{ij} + \Phi_0^{(ij)} I_{ij}^{-1})$.

σ_{ij}^2 follows inverse gamma distribution having form,

$$\sigma_{ij}^2 | Y, \Phi^{ij}, r_{ij}, d_i, T_B \sim IG \left(\frac{n_{ij} + p_{ij} + 1}{2} + \mu_{ij}, S_{ij} \right) \dots (1.7)$$

where,

$$S_{ij} = \frac{1}{2} \left[(Y_{ij} - \Phi^{(ij)} X_{ij})' (Y_{ij} - \Phi^{(ij)} X_{ij}) + (\Phi^{(ij)} - \Phi_0^{(ij)})' I_{ij}^{-1} (\Phi^{(ij)} - \Phi_0^{(ij)}) + 2v_{ij} \right]$$

The uncertain posterior distribution of r_{ij} is

$$\pi(r_{ij} | Y, \Phi^{ij}, \sigma_{ij}^2, d_i, T_B) \propto (\sigma_{ij}^2)^{-\frac{n_{ij}}{2}} \exp \left[- \frac{1}{2\sigma_{ij}^2} (Y_{ij} - \Phi^{(ij)} X_{ij})' (Y_{ij} - \Phi^{(ij)} X_{ij}) \right] \dots (1.8)$$

The uncertain posterior distribution of d_i is a multinomial distribution with probability mass function is

$$P(d_i | Y, \Phi^{ij}, \sigma_{ij}^2, r_{ij}, T_B) = \frac{L(\Theta | Y)}{\prod_{d_i=1}^{d_{0i}} \pi(\Theta | Y)} \dots (1.9)$$

The uncertain posterior distribution of T_B is

$$P(T_B | Y, \Phi^{ij}, \sigma_{ij}^2, r_{ij}, d_i) = \frac{P(T_B, Y | \Phi^{ij}, \sigma_{ij}^2, r_{ij}, d_i)}{P(Y | \Phi^{ij}, \sigma_{ij}^2, r_{ij}, d_i)} \dots (1.10)$$

where,

$$P(T_B, Y | \Phi^{ij}, \sigma_{ij}^2, r_{ij}, d_i) = \frac{\prod_{i=1}^m d_{i0}^{-1} \prod_{i=1}^m \prod_{j=1}^{k_i} \frac{(2\pi)^{-\frac{n_{ij}}{2}} v_{ij}^{\mu_{ij}} \Gamma\left(\frac{n_{ij}}{2} + \mu_{ij}\right)}{(b_{ij} - a_{ij}) \Gamma(\mu_{ij}) |X'_{ij} X_{ij} + I_{ij}^{-1}|^{\frac{1}{2}}}}{\left[\frac{1}{2} \left\{ Y'_{ij} Y_{ij} + \Phi_0^{(ij)'} I_{ij}^{-1} \Phi_0^{(ij)} - (X'_{ij} Y_{ij} + \Phi_0^{(ij)} I_{ij}^{-1})' (X'_{ij} X_{ij} + I_{ij}^{-1})^{-1} (X'_{ij} Y_{ij} + \Phi_0^{(ij)} I_{ij}^{-1}) + 2v_{ij} \right\} \right]}$$

$$P(Y | \Phi^{ij}, \sigma_{ij}^2, r_{ij}, d_i) = \sum_{T_1=1}^{T-m} \dots \sum_{T_m=T_{m-1}+1}^{T-1} P(T_B, Y | \Phi^{ij}, \sigma_{ij}^2, r_{ij}, d_i).$$

From the above equations we observe that the conditional prospective populations of all other parameter, with the exception of targeted treatments and break sites, took the shape of a frequency distribution table. The randomly wander Metropolis Hilton (M-H) technique is used to estimate the quell form features for analytical purposes. Yet we can use the Gibbs free sample approach to get estimate for the unconditional majority of certain parameters with a template dispersion. Several loss functions that describe the minimal average risk connected to an estimator might be taken into consideration for improved estimated selection. Hence, in order to draw more accurate conclusions regarding the model's properties, we used SELF, ALF, and LLF.

Simulation Study

This article describes a simulation study for the suggested model to show how well predictors work in a Bayesian scenario. The specific result regimes TAR system with two match points is taken into consideration for creating a synthetic series. The duration of something like the series is T=600, and breaks occur at life stages T 1=200 and T 2=400. The equation thus describes the presumptive simulation results with starting actual hyperparameters (1.11)

$$y_t = \begin{cases} \begin{cases} 0.20 + 0.20y_{t-1} + 0.2e_t^{(11)} & \text{if } y_{t-1} \leq 0.4 \\ 0.10 + 0.20y_{t-1} + 0.3e_t^{(12)} & \text{if } y_{t-1} > 0.4 \end{cases} & \text{for } 1 \leq t \leq 200 \\ \begin{cases} 0.25 + 0.15y_{t-1} + 0.25e_t^{(21)} & \text{if } y_{t-1} \leq 0.2 \\ 0.15 + 0.20y_{t-1} + 0.40e_t^{(22)} & \text{if } y_{t-1} > 0.2 \end{cases} & \text{for } 200 < t \leq 400, \dots (1.11) \\ \begin{cases} 0.15 + 0.10y_{t-1} + 0.20e_t^{(31)} & \text{if } y_{t-1} \leq 0.5 \\ 0.20 + 0.25y_{t-1} + 0.35e_t^{(32)} & \text{if } y_{t-1} > 0.5 \end{cases} & \text{for } 400 < t \leq 600 \end{cases}$$

where,

$\{e_t^{ij}, i = 1,2,3; j = 1,2\}$ is produced from normal distribution w.r.t true alteration $\sigma_{ij}^2 = ((0.2,0.3); (0.25,0.4); (0.2,0.35))$.

Also, we'll presume that the parameters of the model are present and accept answers here.

$\Phi_0^{(ij)} = (0,0), \mu_{ij} = 2, \nu_{ij} = 1, d_{i0} = 3, a_{ij} = p_{i,5}$ and $b_{ij} = p_{i,95}$, where p_{ik} denotes k^{th} percentile of i^{th} break interval series (T_{i-1}, T_i) .

And use the Gibbs sampler and M-H method approaches, we obtain 1000 downstream observations from the contingent anterior range for estimate and throw away that the very first 200 iteration as a burn-in phase. In order to determine the posterior distribution of the position of the match points there under assumption that the numbers of countries and the quantity of breaches are specified, we first mimic two breaks using a multi ruler TAR(1) series. Figure 4.10 shows that the time points $T_{_1}=205$ and $T_{_2}=398$ are when the conditioned conditional distribution of the breaking factor reaches its greatest likelihood. This leads to the conclusion that the basic probability of breaking location roughly pinpoints when fundamental fractures occurs there in TAR model.

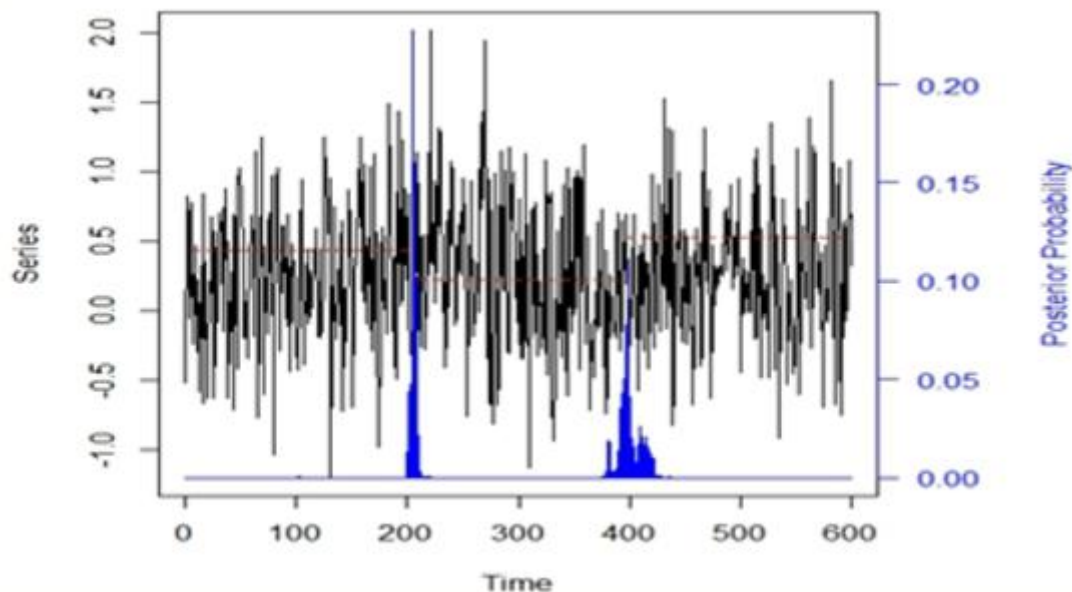


Figure: Modelled sequences with predicted discontinuities

Conclusion

In order to analyse the TAR model with numerous structural breakdowns across several regimes, a Bayesian framework is presented in this chapter. Breakpoint detection and MB-TAR parameter estimation are the two main components of this approach. Gibbs sampling and the M-H method are used to get an approximation of the conditional posterior distribution, which is then used to estimate the unknown parameters. The suggested Bayesian setup accurately estimates model parameters and finds their break points based on the numerical example

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