International Journal of Research in IT and Management (IJRIM)
Available online at: http://euroasiapub.org
Vol. 13 Issue 12, December- 2023
ISSN(o): 2231-4334 | ISSN(p): 2349-6517 | Impact Factor: 8.106

A STUDY OF LOGICAL REASONING IN ABSTRACT ALGEBRA

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ABSTRACT

As more and more applications of its ideas are discovered, abstract algebra has surged in popularity as an area of study in mathematics education. Wasserman demonstrated, for instance, how teaching abstract algebra radically altered both teacher and student perspectives and methods. The students said that their perspectives on elementary and secondary mathematics, particularly arithmetic and algebra, shifted after studying group theory. This result was echoed in a research by Even that used identical methods. An examination of the potential connections between the mathematics taught in an abstract algebra course for working educators and other parts of their field's curricular framework for the teaching of that subject. Teachers said that abstract algebra helped them develop a deeper grasp of mathematics and, on a more epistemological level, that it changed their perspective on the nature of mathematics and what it means to perform mathematics. Although there are many advantages to becoming a teacher, this does not always mean that these advantages will be passed on to pupils. Several studies confirm that teachers' familiarity with abstract algebra has little impact on their students' performance in the classroom. With this tension comes the question of whether or whether students, as opposed to merely professors, can gain from exposure to abstract algebra.

KEYWORDS: Logical Reasoning, Abstract Algebra, mathematics education

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INTRODUCTION

In the field of abstract algebra, the operations between the elements of a set, the mappings between distinct sets, and the mappings defined from a set to itself are all covered. Anything, even other sets, may be included in a set's elements. What they are might be anything from integers to complex polynomials to 'variables' and so on. It's also possible that they're the building blocks of reasoning, which is why this study's inquiry is so important. Finally, we are defining some of the cell's unidentified components as members of a set. You may find yourself thinking of the cell itself as a set. To begin with, we are working with much smaller sets like the four DNA bases, which we may refer to as the alphabet of DNA. This is not necessarily incorrect. For the remainder of this part, provides an overview of abstract algebra's fundamental concepts.

A. Applications to Network Coding

Our goal is not to summaries 40-year-old findings so that they might be used in the way they were originally intended. We merely need to give bibliographical references in order to do this. When it comes to the development of bioinspired computer research, the purpose of all this algebra is not functionalist (i.e. improving performance), but rather to understand the algebraic structure itself and observe how it relates to logic on one hand and DNA on the other. The biological insights are expected to deliver a variety of benefits, including improved performance, but the first goal is to build a common formalism for these very disparate areas.. It's for this reason that the "case study" of network coding is so helpful since it gives a very real-world context for the abstract notions being covered. To a lesser degree, the following discussion depends on.

Linear block codes are a strong algebraic structure that we may build upon by gradually adding new restrictions. To begin, consider a vector space. A vector space is a field with more than one dimension, even though this definition is a little too basic. A tuple of n objects, each of which is an element of a field F, is an element of a vector space of dimension n. F n is a symbol for an n-dimensional vector space over F. Binomial field Z2 (F) contains precisely two elements when F is represented by the binary field Z2. Adding redundancy to a message allows it to be reconstructed in the event of transmission problems, which is why linear block codes are used. Breaking up a signal stream into k message blocks is done by taking an indeterminate number of messages from F k and dividing them by k. As a result, the message space is a k-dimensional vector space with precisely 2 k items. (message blocks). Unambiguous and invertible coding requires a 1-1 mapping between the message space and code space, which is only possible if the mapping is inverted. A subspace of F n with dimension k comprises 2 k separate components, each longer than dimension k, making up the code space. The encoding map becomes a linear transformation (i.e. a matrix multiplication) as a result of this arrangement ([12], 239). Using a matrix of dimensions k n, each message block (row vector) is multiplied by the matrix (k rows by n columns). Each codeword is a code vector of length n, which is the result of this multiplication for each block. Using a wireless connection, the transmitter sends each codeword and the receiver decodes it. Each codeword must be corrected and mapped back to the original message block in order for it to be reconstructed.

B. an Observation on Gordon’s Method

For BCH codes, Gordon's approach deals with determining the minimal polynomials of roots in an extension field GF (2e). They may be described in terms of the GF (2) [x]/[f(x)] factor rings, or as successive powers of one of the basic elements, or as the remainders of the irreducible primitive polynomial f(x), which is generated by the extension field GF (2e). These lowest polynomials have 2 e roots because their degree may be as high as e in any given extension fielded. Each minimal polynomial may also be represented modulo the same basic polynomial, which Gordon realized. To put it another way, a minimal polynomial may be written as an element from its own roots. For low-power space probes, this makes it trivially straightforward to determine the minimum polynomials, which are excellent for low memory and CPU cycles. A minimal polynomial and one of its roots could actually have the same polynomial form, which is even more astonishing! Although the relevance of this phenomenon is unclear, we can't help but be intrigued by instances of structural invariance like this. To a lesser degree, it is evocative to the ideas of eigenvector in oscillatory systems or the renormalization group in statistical mechanics, but it may be explained by delving further into Galois theory. It may first seem that biology and algebra are completely unrelated, however in this section we have utilised network coding to offer a taste of the richness of algebraic structure. As it turns out, the research described in shows the opposite. In the meanwhile, let's have a look at the structure of logic.

PRELIMINARIES

Finding and studying applications of abstract algebra requires a certain level of mathematical maturity. Set theory, mathematical induction, equivalence relations, and matrices are all necessary but not sufficient prerequisites. The ability to read and comprehend mathematical proofs is much more crucial. An abstract algebra course needs a solid foundation in mathematics.

A Short Note on Proofs

Mathematical abstraction is distinct from other fields of study. Scientists in fields like chemistry and physics who work in the lab conduct experiments to help them find and validate hypotheses. Despite the fact that mathematics is frequently based on physical experiments or computer simulations, it is rigorous since it is based on logical reasons. When studying abstract mathematics, we use a method known as an axiomatic approach, which entails assuming certain principles regarding the structure of a set of objects called S. They're known as axioms. We'd want to find out more about S by applying logical reasoning to the axioms we've established for it. Our axioms must be consistent, which means they cannot be in conflict with one another. Additionally, we insist that there be not too many axioms. Few instances of mathematical structures may be found when an axiom system is overly restrictive.

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A logical or mathematical statement is an assertion that can be shown to be true or wrong. Consider the following examples as a guide:

- 3 + 56 − 13 + 8/2.
- All cats are black.
- 2 + 3 = 5.
- 2x = 6 exactly when x = 4.
- If ax2 + bx + c = 0 and a 6= 0, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- X 3 − 4x 2 + 5x − 6.

Statements may only be true or false; the first two instances are exceptions.

When it comes to mathematics, all it takes to prove anything is a strong argument that the proposition in question is true. That assertion is wrong because assessing 2 • 4 and noting that 6 6=8 shows that the statement "2x = 6 precisely when x equals 4" is untrue. This kind of argument should include enough information to persuade the listeners. Proofs might be directed to a student, a professor, or a reader of a work. Of course, audiences can vary greatly. The explanation will either be lengthy or badly worded if more information is included than is necessary in the evidence. An insufficient amount of information may render the evidence ineffective. Keep the audience in mind once again. Compared to graduate students, high school kids need a lot more information. When writing an argument in abstract algebra class, it's excellent practise to write it to persuade one's peers, regardless of whether those peers are other students or readers of the book.

Let's take a look at a few different kinds of claims. Mathematicians, on the other hand, are more interested in assertions like "If p, then q," where p and q are both propositions, than in simple statements like "10/5 = 2.". We want to know what we can say about other assertions if some statements are known or presumed to be true. The hypothesis is referred to as p, while the conclusion is referred to as q. Take a look at the following: Assuming ax2 + bx + c = 0, and as a result, a 6 = 0, then,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Assumption and conclusion are the same, and the hypothesis is that Ax2 + Bx + C = 0.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Notice that the statement does not address the validity of the hypothesis. It is possible to prove that the conclusion is correct if this complete statement is accurate and we can prove that ax2+bx+c=0 with a 6=0 is true. A simple sequence of equations might be used to prove this claim:

$$\begin{aligned} ax^2 + bx + c &= 0 \\ x^2 + \frac{b}{a}x &= -\frac{c}{a} \\ x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 &= \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \end{aligned}$$

A proposition is a statement that can be proven to be true. A theorem is the scientific term for a significant proposition. A proof may be broken down into modules instead of being proven as a whole; that is, we can establish multiple supporting propositions (known as lemmas) and then utilise the findings of these propositions to prove our primary conclusion. If we can demonstrate an assertion or a theorem, we may typically deduce similar propositions called corollaries with minimal effort.

1 Some Cautions and Suggestions

There are a variety of ways to prove a claim. There are many frequent errors that students make while learning how to prove theorems, in addition to using various proof techniques. For the benefit of students learning abstract mathematics for the first time, we've compiled a list of common problems and their solutions. It's a good idea to revisit this list from time to time as a refresher course. This chapter and the rest of the material will provide more methods of proving a point.

- A theorem cannot be proven by example; nonetheless, the conventional method of proving that a statement is not a theorem is to present a counterexample.

- The use of quantifiers is critical. There are many diverse meanings to words and phrases like "only,""for all,""for every," and "for some.

- If a hypothesis isn't clearly mentioned in a theorem, don't assume it. Things can't be assumed.

- So, you want to prove the existence of a certain item and its uniqueness. The first step is to prove that the thing exists. In order to demonstrate that it is unique, suppose that there are two such objects, say r and s, and then prove that r = s.

- Proving the opposite of a statement is sometimes simpler. Isn't it just as easy to prove that something is true "if" something else is true "if not" anything else?

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○ It is preferable to discover direct evidence; however this may be difficult in certain cases. Assuming the proposition you're attempting to disprove is incorrect and hoping you'll be forced to make an absurd remark in the course of your argument may be an easier option.

Keep in mind that proving theorems is an important part of advanced mathematics. Theorems are mathematical tools that enable novel and fruitful applications of mathematics. “We utilise examples to explain current theorems and to help students develop their own ideas about what could be true in the future. The connections between applications, examples, and proofs are much stronger than they appear at first glance.

2 Sets and Equivalence Relations

Set Theory

A set is a collection of items whose definition is such that it is possible to identify whether or not any given object x is a member of the set. Elements and members are the terms used to describe the things that make up a set. When referring to sets, we'll use capital letters, such as A or X. For example, we'll write an A to show that an is a member of set A.

An object x may be excluded from a set by specifying an attribute that defines whether or not it is a member of the set, or by listing all of the items of the set. Alternatively, we may compose a piece of writing.

X = {x1, x2. . . xn}

In the case of a set having the entries (x1-xn)

X = {x: x satisfies P}

Then P is true for every x in X. By writing either "E" or "E," we may express the set of even positive integers.

E = {2, 4, 6 . . .} or E = {x: x is an even integer and x > 0}.

As an example, we write 2 / E to indicate that 2 is not in the set E, and we write 2 in the same way to indicate that 2 is included in the set E.

We'll take a look at a few of the most essential sets:

N = {n: n is a natural number} = {1, 2, 3 . . .};

Z = {n: n is an integer} = {. . . , −1, 0, 1, 2 . . .};

Q = {r: r is a rational number} = {p/q: p, q ∈ Z where q 6= 0};

R = {x: x is a real number};

C = {z: z is a complex number}.

We may execute operations on sets and discover different relationships between them. Assuming that every member of A is also an element of B, a set A is a subset of B. For instance,

{4, 5, 8} ⊂ {2, 3, 4, 5, 6, 7, 8, 9}

And

N ⊂ Z ⊂ Q ⊂ R ⊂ C

Every set is a subset of itself, which is a trivial fact. As long as B is less than or equal to A, but B 6 is equal to A, it is a valid subset of A. To express the fact that a set isn't a subset of another, we write A 6 B. For example, A 6 4, 7, 9 In order for two sets to be considered equal, we must be able to prove that one set is a subset of the other.

Having a set with no components is handy. denotes this set, which is known as the empty set. The empty set is a subset of all sets.

To generate new sets out of existing sets, we may do specific operations: It is said that the

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union A B of two collections A and B is

$$A \cup B = \{x: x \in A \text{ or } x \in B\};$$

The intersection of A and B is defined by

$$A \cap B = \{x: x \in A \text{ and } x \in B\}.$$

If A = {1, 3, 5} and B = {1, 2, 3, 9}, then

$$A \cup B = \{1, 2, 3, 5, 9\} \text{ and } A \cap B = \{1, 3\}.$$

There are several ways to think about union and intersection. As a result of this, we write

$$\bigcup_{i=1}^{n} A_i = A_1 \cup \ldots \cup A_n$$

And

$$\bigcap_{i=1}^{n} A_i = A_1 \cap \ldots \cap A_n$$

For the sets A1,..., An. are the union and intersection, respectively.

E and O, the sets of even and odd numbers, are disjoint because they have no shared members. When A B =, the sets A and B are no longer connected.

The universal set is a fixed set of values that we can deal with at times. We define A0, the complement of A, as the set A for every set A U.

$$A\ 0 = \{x: x \in U \text{ and } x\ /\!\in A\}.$$

The difference between two sets A and B is referred to as

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A \ B = A ∩ B 0 = {x: x ∈ A and x /∈ B}.

Example 1 Let R be the universal set and suppose that

A = {x ∈ R: 0 < x ≤ 3} and B = {x ∈R : 2 ≤ x < 4}.

Then

$$\begin{aligned} A \cap B &= \{x \in \mathbb{R} : 2 \leq x \leq 3\} \\ A \cup B &= \{x \in \mathbb{R} : 0 < x < 4\} \\ A \setminus B &= \{x \in \mathbb{R} : 0 < x < 2\} \\ A' &= \{x \in \mathbb{R} : x \leq 0 \text{ or } x > 3\}. \end{aligned}$$

Proposition 1.1 Let A, B, and C be sets. Then

1. A ∪ A = A, A ∩ A = A, and A \ A = ∅;

2. A ∪∅ = A and A ∩ ∅ = ∅;

3. A ∪ (B ∪ C) = (A ∪ B) ∪ C and A ∩ (B ∩ C) = (A ∩ B) ∩ C;

4. A ∪ B = B ∪ A and A ∩ B = B ∩ A;

5. A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C);

6. A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).

Proof We will prove (1) and (3) and leave the remaining results to be proven in the exercises.

(1) Observe that

$$\begin{aligned} A \cup A &= \{x : x \in A \text{ or } x \in A\} \\ &= \{x : x \in A\} \\ &= A \end{aligned}$$

And

$$\begin{aligned} A \cap A &= \{x : x \in A \text{ and } x \in A\} \\ &= \{x : x \in A\} \\ &= A. \end{aligned}$$

Also, A \ A = A ∩ A0 = ∅.

(3) For sets A, B, and C,

$$\begin{aligned} A \cup (B \cup C) &= A \cup \{x : x \in B \text{ or } x \in C\} \\ &= \{x : x \in A \text{ or } x \in B, \text{ or } x \in C\} \\ &= \{x : x \in A \text{ or } x \in B\} \cup C \\ &= (A \cup B) \cup C. \end{aligned}$$

A similar argument proves that A ∩ (B ∩ C) = (A ∩ B) ∩ C.

Theorem 1.1 (De Morgan's Laws) Let A and B be sets. Then

1. (A ∪ B) 0 = A0 ∩ B0;

2. (A ∩ B) 0 = A0 ∪ B0.

Proof The first step is to prove that (A – B) (0 – A0 – B0) and (A – B) (0 – A0 – B0) are the same. A is equal to B, thus we'll set the limit to 0. x = A + B then. As a result, by the definition of the union of sets, x does not belong in either A nor B. x A0 and x B0 are the complements of each other. As a result, x A0 B0 and (A B) 0 A0 B0 are obtained.

Assume that x is a subset of A0 that is included in B0. There is no need to worry about whether or not x is more than A0 or greater than B0; it is equal to A0 and B0. As a result, x = A B and x = (A B) 0. Because (A B) 0 = A0 B0, (A B) 0 = A0 B0, we conclude

The proof of (2) is left as an exercise.

Example 2 Other relations between sets often hold true. For example,

$$(A \setminus B) \cap (B \setminus A) = \emptyset.$$

To see that this is true, observe that

$$\begin{aligned}(A \setminus B) \cap (B \setminus A) &= (A \cap B') \cap (B \cap A') \\ &= A \cap A' \cap B \cap B' \\ &= \emptyset.\end{aligned}$$

CONCLUSION

Logically-based verbal models contain a wealth of information that may be used to formulate a mathematical solution to the problem. This implies that every so often, abstract algebra is used to translate figurative language into a form that the technique can work with to create the best possible or, at the very least, the most usable structure. Reading any book on mathematics, management, or science will always lead to the discovery of a few problems presented in written form. The goal of the investigation is to formulate the question as a mathematical problem to which a solution may be found. Many of the factors that influence students' performance in relation to goals stated by digital evidence might be addressed with a new requirement, so long as the same impacts (or lack thereof) are noticed in all settings. Students' increasing reliance on logical superstitions is a surefire recipe for professional failure; yet, even the most recent proof or supporters for these ideas sometimes relied on suspect math.

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