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HIGHER ORDER NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

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Abstract

This study examines recent advances in computing techniques for solving nonlinear ordinary differential equations (ODEs), with a focus on the Natural Transform Method (NTM) and its applications. The article introduces the Natural Decomposition Method (NDM) as a novel strategy for resolving nonlinear ODEs and provides a comprehensive description of the NTM, including its integral transforms such as the Laplace and Sumudu transforms. Using NTM and a series solution architecture that incorporates Adomian polynomials and recursive relations, the NDM may obtain precise or approximate solutions for complex differential equations. The value of these techniques is demonstrated by worked examples that demonstrate how solutions produced using the NDM substantially resemble known results.

Keywords: Natural Transform Method, Computational, Techniques, Ordinary Differential Equations

1. INTRODUCTION

Nonlinear ordinary differential equations, or ODEs, are crucial to understanding and simulating complex behaviors in a variety of scientific and engineering fields, such as physics, biology, chemistry, economics, and engineering. Unlike linear differential equations, nonlinear ODEs involve complicated interactions where the dependent variable and its derivatives show nonlinear behavior. This nonlinearity often leads to complex dynamics such as bifurcations, chaos, and other phenomena that cannot be explained by linear systems. Because of this, solving nonlinear ODEs analytically is often impractical, necessitating the development of complex computational techniques to generate approximations with a respectable degree of accuracy. Conventional numerical methods such as the Euler method, Runge-Kutta methods, and finite difference approaches have historically been used to solve nonlinear ODEs [1]. Despite the fact that these methods have proved effective in solving a variety of problems, their application is usually limited by challenges with accuracy, stability, and convergence—especially when dealing with stiff equations or extremely nonlinear systems. These limitations have led to a lot of research and innovation in computational mathematics, which has produced ever-more-complex techniques for handling the complexity of nonlinear ODEs. Recent advancements in machine learning-based methods, particularly neural networks, have opened up a new paradigm for solving nonlinear ODEs. These techniques approximate solutions by learning complex patterns and correlations from training data using the data-driven architecture of neural networks [2]. In addition, met heuristic computations such as Differential Advancement (DE), Molecule Multitude Improvement (PSO), and Hereditary Calculations (GA) have been used with conventional mathematical methods to increase arrangement accuracy and registering productivity. These half-and-half approaches combine the benefits of both deterministic and stochastic approaches to provide stable arrangements even in situations with complicated limit conditions or multifaceted nonlinear frameworks. The growing need for accuracy and efficiency in solving nonlinear ODEs has also spurred the development of adaptive and multistate techniques. By constantly modifying discretization and processing resources in accordance with the local aspects of the problem, these techniques ensure that the numerical solutions are both computationally viable and accurate. Additionally, techniques such as spectral and finite element methods have been refined to better handle the challenges posed by nonlinearities and stiff equations. The work of partial math in enhancing the accuracy and efficiency of mathematical arrangements is highlighted by Ahmad et al. (2020), who explore novel perspectives on regular solutions for nonlinear time fragmented halfway differential equations. Their review provides creative approaches to managing nonlinearities through the use of partial request administrators, providing a more comprehensive system for illustrating memory-impacted operations. In contrast to traditional methods, this work demonstrates how fragmented math-based tactics may be used to a wide range of nonlinear frameworks, providing improved intermingling and security. The latest developments in computational fluid dynamics (CFD) and how they are used to solve nonlinear differential equations are the main topics of Bhatti et al. (2020). Their study demonstrates how sophisticated numerical and simulation approaches are increasingly being used to represent intricate fluid flows controlled by nonlinear ODEs and PDEs. The authors talk about how CFD models' predictive power has been greatly increased by integrating them with contemporary computational tools like machine learning and data-driven methodologies. This paper highlights the significance of using hybrid approaches that mix deterministic and stochastic methodologies for solving nonlinear fluid dynamics problems by addressing issues with accuracy, stability, and convergence.

2. FUNDAMENTAL THOUGHT OF THE NORMAL CHANGE TECHNIQUE

We give some foundation data with respect to the idea of the Normal Change Strategy (NTM) in this part. Given a capability $f(t)$, where $t \in (-\infty, \infty)$, the overall basic change has the accompanying definition:

$$\Im[f(t)](s) = \int_{-\infty}^{\infty} K(s,t) f(t) dt, \quad (1)$$

where $K(s,t)$ means the change's piece and s is a genuine, complex number that is free of t. Remember that tJn(st) and t when K(s, t) is e⁻s t^{s-1} (st), then Equation (1) gives the Hankel change, Mellin change, and Laplace change, in a specific order change. Let us now investigate the integral transformations described by for $(t), t \in (-\infty, \infty)$:

$$\Im[f(t)](u) = \int_{-\infty}^{\infty} K(t) f(ut) dt$$
$$\Im[f(t)](s,u) = \int_{-\infty}^{\infty} K(s,t) f(ut) dt \quad (2)$$

It is vital to take note of that, when K(t) = e − t, the fundamental Sumudu change is given by Equation (2), where u is utilized instead of boundarys. Moreover, the summed-up Laplace and Sumudu changes are characterized, individually for any worth of:

$$\ell[f(t)] = F(s) = s^n \int_0 e^{-s^{n+1}t} f(s^n t) dt$$
$$\mathbb{S}[f(t)] = G(u) = u^n \int_0^{\infty} e^{-u^n t} f(tu^{n+1}) dt \quad (3)$$

Observe that the Laplace and Sumudu changes, individually, are addressed by Equations (2) and (3) when n = 0. The capability $f(t)$ for $t \in (-\infty, \infty)$ has the accompanying regular change characterized by:

$$\mathbb{N}[f(t)] = R(s,u) = \int_{-\infty}^{\infty} e^{-st} f(ut)dt; s,u \in (-\infty,\infty) \quad (4)$$

where the factors s and u are the regular change factors and $N[f(t)]$ is the time capability $f(t$'s) regular change [5-8]. Remember that Eq. (4) can be communicated as:

$$\begin{aligned}\mathbb{N}[f(t)] &= \int_{-\infty}^{\infty} e^{-st} f(ut)dt; s,u \in (-\infty,\infty) \\ &= \left[\int_{-\infty}^{0} e^{-st} f(ut)dt; s,u \in (-\infty,0)\right] + \left[\int_{0}^{\infty} e^{-st} f(ut)dt; s,u \in (0,\infty)\right] \\ &= \mathbb{N}^-[f(t)] + \mathbb{N}^+[f(t)] \\ &= \mathbb{N}[f(t)H(-t)] + \mathbb{N}[f(t)H(t)] \\ &= R^-(s,u) + R^+(s,u)\end{aligned} \quad (5)$$

The Heaviside capability is addressed by H(.). This ought to be noted: if the capability $f(t)H(t)$ is characterized on the positive genuine hub for all t values in R, then, at that point, the Regular change (N-Change) is characterized on the set the Heaviside capability is addressed by H(.). Remember that Equation (6) can be diminished to the Sumudu change if s=1 and to the Laplace change if u=1. We currently give a portion of the N Changes alongside their transformation to Laplace and Sumudu.

$$A = \left\{ \begin{matrix} f(t): \exists M, \tau_1, \tau_2 > 0, \text{ such that } |f(t)| < Me^{\frac{|t|}{\tau_j}}, \\ \text{if } t \in (-1)^j \times [0,\infty), j \in \mathbb{Z}^+ \end{matrix} \right\} \quad (6)$$

$$\mathbb{N}[f(t)H(t)] = \mathbb{N}^+[f(t)] = R^+(s,u) = \int_{0}^{\infty} e^{-st} f(ut)dt; s,u \in (0,\infty)$$

Table 1: Unique N-Transforms and the transformation to Laplace and Sumudu

$f(t)$	$\mathbb{N}[f(t)]$	$\mathbb{S}[f(t)]$	$\ell[f(t)]$
1	$\frac{1}{s}$	1	$\frac{1}{s}$
t	$\frac{u}{s^2}$	u	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-au}$	$\frac{1}{1-au}$	$\frac{1}{s-a}$
$\frac{t^{n-1}}{(n-1)!}, n=1,2,\ldots$	$\frac{u^{n-1}}{s^n}$	u^{n-1}	$\frac{1}{s^n}$
$\sin(t)$	$\frac{u}{s^2+u^2}$	$\frac{u}{1+u^2}$	$\frac{1}{1+s^2}$

Table 2: Properties of N-Transforms

Functional Form	Natural Transform
$y(t)$	$Y(s,u)$
$y(at)$	$\frac{1}{a}Y(s,u)$
$y'(t)$	$\frac{s}{u}Y(s,u)-\frac{y(0)}{u}$
$y''(t)$	$\frac{s^2}{u^2}Y(s,u)-\frac{s}{u^2}y(0)-\frac{y'(0)}{u}$
$\gamma y(t) \pm \beta v(t)$	$\gamma Y(s,u) \pm \beta V(s,u)$

3. Methodology:

Analyze the accompanying general nonlinear ordinary differential condition:

$$Lv + R(v) + F(v) = g(t) \qquad (7)$$

based on the original condition

$$v(0) = h(t) \qquad (8)$$

where g(t) is the nonhomogeneous term, F(v) is the nonlinear term, R is the differential administrator's lingering, and L is the administrator of the greatest subordinate. Assuming L is a first-order differential operator, we can obtain the following by using the N − Transform of Equation (7):

$$\frac{sV(s,u)}{u} - \frac{V(0)}{u} + \mathbb{N}^{+}[R(v)] + \mathbb{N}^{+}[F(v)] = \mathbb{N}^{+}[g(t)] \qquad (9)$$

When we change Eq. (8) to Eq. (9), we get:

$$V(s,u) = \frac{h(t)}{s} + \frac{u}{s}\mathbb{N}^{+}[g(t)] - \frac{u}{s}\mathbb{N}^{+}[R(v) + F(v)] \qquad (10)$$

By taking the N -Change of Condition (10) contrarily, we acquire:

$$v(t) = G(t) - \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}[R(v) + F(v)]\right] \qquad (11)$$

where the source term is G(t). Now, we'll assume that the unknown function $v(t)$ of the following form has an infinite series solution:

$$v(t) = \sum_{n=0}^{\infty} v_n(t) \qquad (12)$$

Next, we can rewrite Eq. (11) in the following form by applying Eq. (12):

$$\sum_{n=0}^{\infty} v_n(t) = G(t) - \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}[R\sum_{n=0}^{\infty} v_n(t) + \sum_{n=0}^{\infty} A_n(t)]\right] \qquad (13)$$

where the nonlinear term is represented by the Adomian polynomial An(t). We can quickly construct the recursive relation by comparing the two sides of Eq. (13) as shown below:

$$\begin{aligned} v_0(t) &= G(t) \\ v_1(t) &= -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^+[Rv_0(t)+A_0(t)]\right] \\ v_2(t) &= -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^+[Rv_1(t)+A_1(t)]\right] \\ v_3(t) &= -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^+[Rv_2(t)+A_2(t)]\right] \end{aligned} \quad (14)$$

We eventually have the broad recursive relation shown here:

$$v_{n+1}(t) = -\mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^+[Rv_n(t)+A_n(t)]\right],\ n \geq 0 \quad (15)$$

Therefore, the exact or approximative answer is provided by:

$$v(t) = \sum_{n=0}^{\infty} v_n(t) \quad (16)$$

Example: Look at the accompanying first-request nonlinear differential condition:

$$\frac{d^2v}{dt^2} + \left(\frac{dv}{dt}\right)^2 + v^2(t) = 1 - \sin(t) \quad (17)$$

based on the original condition

$$v(0) = 0,\ v'(0) = 1 \quad (18)$$

To begin with, we apply the N -change to the two sides, and the outcome is:

$$\frac{s^2V(s,u)}{u^2} - \frac{sV(0)}{u^2} - \frac{v'(0)}{u} + \mathbb{N}^+\left[\left(\frac{dv}{dt}\right)^2\right] + \mathbb{N}^+[v^2(t)] = \frac{1}{s} - \frac{u}{s^2+u^2} \quad (19)$$

We get the following by changing Eq. (18) to Eq. (19):

$$V(s,u) = \frac{u^2}{s^3} + \frac{u}{s^2+u^2} - \frac{u^2}{s^2}\mathbb{N}^+\left[\left(\frac{dv}{dt}\right)^2 + v^2(t)\right] \quad (20)$$

Next, using Eq. (20) inverse N-Transform), we obtain:

$$v(t) = \frac{t^2}{2!} + \sin(t) - \mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^+\left[\left(\frac{dv}{dt}\right)^2 + v^2(t)\right]\right] \quad (21)$$

Now, we'll assume that the unknown function $v(t)$ has an infinite series solution of the following form:

$$v(t) = \sum_{n=0}^{\infty} v_n(t) \quad (22)$$

We may rewrite Eq. (21) as follows by utilizing Eq. (22)

$$\sum_{n=0}^{\infty} v_n(t) = \frac{t^2}{2!} + \sin(t) - \mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} B_n\right]\right] \quad (23)$$

where the Adomian polynomials of the nonlinear terms (dv/dt)2 and $v^2(t)$ are, separately, A_n and B_n. From that point forward, we might drive the overall recursive connection as follows by contrasting the different sides of Equation (23):

$$\begin{aligned} v_0(t) &= \frac{t^2}{2!} + \sin(t) \\ v_1(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[A_0 + B_0]\right] \\ v_2(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[A_1 + B_1]\right] \\ v_3(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[A_2 + B_2]\right] \end{aligned} \quad (24)$$

The broad recursive relation is thus provided by

$$v_{n+1}(t) = -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[A_n + B_n]\right],\ n \geq 0 \quad (25)$$

The excess parts of the obscure capability v(t) can subsequently be basically processed as follows utilizing the recursive connection characterized in Eq. (25):

$$\begin{aligned} v_1(t) &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[A_0 + B_0]\right] \\ &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[(v_0')^2 + v_0^2]\right] \\ &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[(v_0')^2 + v_0^2]\right] \\ &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^2}\mathbb{N}^{+}[1]\right] + \cdots \\ &= -\mathbb{N}^{-1}\left[\frac{u^2}{s^3}\right] + \cdots \\ &= -\frac{t^2}{2!} + \cdots \end{aligned} \quad (26)$$

Thusly, it is feasible to exhibit that the non-dropped part of $v_0(t)$ still fulfills the gave differential condition by dropping the clamor terms that arise somewhere in the range of $v_0(t)$ and $v_1(t)$. This prompts a definite arrangement of the accompanying structure:

$$v(t) = \sin(t) \quad (27)$$

The precise answer closely matches the outcome that (ADM) was able to acquire.

$$\frac{dv}{dt} - 1 = v^2(t) \quad (28)$$

subject to the initial condition

$$v(0) = 0 \quad (29)$$

Equation (29), when the Natural transform is applied to both sides, yields:

$$\frac{s}{u}V(s,u) - \frac{1}{u}V(s,u) - \frac{1}{s} = \mathbb{N}^{+}[v^2(t)] \quad (30)$$

Replacing Equation (29)

$$V(s,u) = \frac{u}{s^2} + \frac{u}{s}\left[\mathbb{N}^{+}[v^2(t)]\right] \quad (31)$$

we attain

$$v(t) = t + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[v^2(t)]\right]\right] \quad (32)$$

We now assume that the unknown function v(t) has an infinite solution of the following form:

$$v(t) = \sum_{n=0}^{\infty} v_n(t) \quad (33)$$

We can rewrite Eq. (32) using Eq. (33) as follows:

$$\sum_{n=0}^{\infty} v_n(t) = t + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[\sum_{n=0}^{\infty} A_n(t)\right]\right]\right] \quad (34)$$

we can engender the recursive relative as trails:

$$\begin{aligned} v_0(t) &= t \\ v_1(t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[A_0(t)]\right]\right] \\ v_2(t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[A_1(t)]\right]\right] \\ v_3(t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[A_2(t)]\right]\right] \end{aligned} \quad (35)$$

Consequently, the following represents the general recursive relation:

$$v_{n+1}(t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[A_n(t)]\right]\right],\ n \geq 0 \quad (36)$$

We can quickly calculate the remaining elements of the unknown function v(t) using Eq. (36) as follows:

$$\begin{aligned}
v_1(t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[A_0(t)]\right]\right] = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[v_0^2(t)]\right]\right] \\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[t^2]\right]\right] = \mathbb{N}^{-1}\left[\frac{2u^3}{s^4}\right] = \frac{1}{3}t^3, \\
v_2(t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[A_1(t)]\right]\right] = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[2v_0(t)v_1(t)]\right]\right] \\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[\frac{2t^4}{3}\right]\right]\right] = \mathbb{N}^{-1}\left[\frac{48u^5}{3s^6}\right] = \frac{2t^5}{15} \qquad (37) \\
v_3(t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[A_2(t)]\right]\right] = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}[2v_0(t)v_2(t) + v_1^2(t)]\right]\right] \\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[\frac{17t^6}{45}\right]\right]\right] = \mathbb{N}^{-1}\left[\frac{12240u^7}{45s^8}\right] = \frac{17t^7}{315}
\end{aligned}$$

Then the approximate explanation of the unidentified function $v(t)$ is specified by:

$$\begin{aligned}
v(t) &= \sum_{n=0}^{\infty} v_n(t) \\
&= v_0(t) + v_1(t) + v_2(t) + v_3(t) + \cdots \\
&= t + \frac{1}{3}t^3 + \frac{2t^5}{15} + \frac{17t^7}{315} + \cdots
\end{aligned}$$

Now we get

$$v(t) = \tan(t)$$

The precise answer closely matches the outcome that (ADM) was able to acquire.

4. CONCLUSION

The advancements in computer algorithms for solving nonlinear ordinary differential equations (ODEs) show the great potential of the Natural Transform Method (NTM) and related approaches. For the purpose of solving nonlinear ODEs, the NTM offers a solid basis. It contains a number of integral transforms, such as the Sumudu, Laplace, and their generalised forms. By employing these transformations, the Natural Decomposition Method (NDM), which combines Adomian polynomials with recursive relations, provides a systematic approach to solving nonlinear differential equations through a series solution strategy. The examples provided demonstrate how the NDM can be used in practice to obtain exact or nearly exact results that closely resemble existing solutions. By increasing the accuracy of solutions to challenging differential equations and simplifying the computing process, these advancements make it a valuable tool for researchers and practitioners in the field.

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