
INVESTIGATING THE ROLE OF QUEUEING THEORY IN THE MANUFACTURING INDUSTRY

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ABSTRACT

Queueing theory contributes to the optimization of manufacturing processes by analyzing and modelling the movement of entities (materials or tasks) within manufacturing systems. The abstract investigates the significance of queueing theory within the manufacturing industry, with a specific emphasis on how it can enhance efficiency, minimize waiting periods, and decrease resource inefficiencies. The study of queueing theory involves examining variables such as for instance the rate at which people or items turn up, the time it takes to serve them, and the maximum capacity of a system. Its purpose is to pinpoint areas of congestion and devise tactics to improve the efficiency of operations. This summary highlights the valuable knowledge that queueing theory offers to assist in decision-making, improve workflows, and eventually enhance manufacturing performance.

KEYWORDS: - *Queueing theory, Manufacturing processes, Production systems.*

INTRODUCTION

Queueing theory, a branch of operations research and applied mathematics, holds a crucial position in the realm of manufacturing optimization. In manufacturing systems, the flow of entities such as products, materials, and tasks is subject to varying degrees of uncertainty, leading to inefficiencies, delays, and resource underutilization. Queueing theory offers a systematic framework for analyzing and understanding these dynamics, ultimately aiding in the enhancement of production processes.

Manufacturing environments are characterized by intricate interactions among different stages of production, equipment availability, workforce allocation, and task prioritization. These complexities often result in queuing phenomena, where entities wait in line before they can be processed or moved to the subsequent stage. Understanding and managing these queues are essential to minimize production lead times, improve resource allocation, and optimize overall system performance. This paper aims to explore the pivotal role of queueing theory in manufacturing by examining its applications, methodologies, and real-world impact. By analyzing arrival rates, service times, and system capacities, queueing theory enables the identification of operational bottlenecks and the formulation of strategies for more efficient workflows. Additionally, it facilitates the assessment of trade-offs between factors like waiting times, resource utilization, and production costs, enabling manufacturers to make informed decisions. The integration of queueing theory principles into manufacturing practices offers several tangible benefits. These include reduced production downtime, improved customer satisfaction through timely deliveries, better resource allocation leading to cost savings, and

enhanced decision-making based on quantitative insights. As manufacturing processes continue to evolve with technological advancements and increasing demand variability, queueing theory provides a robust foundation for addressing the challenges inherent in such environments.

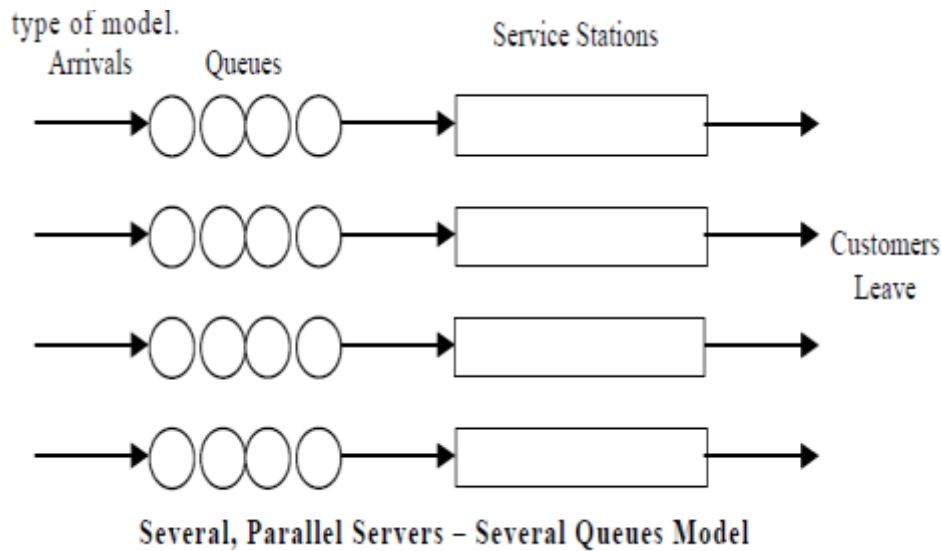


Figure 1 Queueing Theory's flow

In the subsequent sections of this paper, we will delve into the key concepts of queueing theory, its application in various manufacturing scenarios, and case studies showcasing its successful implementation. By shedding light on these aspects, we hope to underscore the transformative potential of queueing theory in revolutionizing manufacturing practices and contributing to operational excellence.

Need of the study

The modern manufacturing landscape is characterized by intricate workflows, dynamic demand patterns, and complex interactions among various production stages. In such a context, efficient resource utilization, minimized wait times, and streamlined processes are paramount for maintaining competitiveness and ensuring customer satisfaction. Queueing theory emerges as a vital tool to address these challenges and optimize manufacturing operations.

Efficiency Enhancement: Manufacturing systems often suffer from inefficiencies due to unbalanced workloads, equipment downtime, and suboptimal resource allocation. Queueing theory's application can uncover hidden bottlenecks and identify areas where resources are underutilized, thus paving the way for more streamlined and efficient processes.

Wait Time Reduction: Excessive wait times, whether for processing or material movement, can lead to increased lead times, delayed deliveries, and dissatisfied customers. Queueing theory provides insights into reducing these wait times by strategically allocating resources, optimizing production schedules, and managing work-in-progress effectively.

Resource Allocation Optimization: Manufacturing operations involve the allocation of

limited resources such as machines, labour, and materials. Queueing theory's mathematical models enable manufacturers to make data-driven decisions about how to allocate these resources to achieve maximum throughput and minimal idle time.

Cost Savings: Inefficient processes often lead to higher operational costs due to overstaffing, overtime expenses, and excess inventory. By analyzing queues and optimizing system parameters, queueing theory can help manufacturers cut down on unnecessary expenses and achieve cost-effective production.

Capacity Planning: Manufacturers need to anticipate fluctuations in demand and ensure that their systems can handle peak loads without compromising efficiency. Queueing theory aids in capacity planning by simulating scenarios, predicting resource requirements, and allowing manufacturers to proactively adjust their operations.

Decision-Making Support: In an environment with multiple variables and trade-offs, informed decision-making is critical. Queueing theory's quantitative models provide a foundation for making strategic choices related to production processes, resource investments, and system improvements.

Technology Integration: With the rise of Industry 4.0 technologies, data-driven insights are becoming increasingly crucial. Queueing theory can be integrated with data analytics and real-time monitoring systems to provide continuous feedback for process improvement and adaptive decision-making.

Continuous Improvement: Manufacturing is an ever-evolving field, and optimizing processes is an ongoing endeavor. Queueing theory provides a structured approach for continuous improvement by allowing manufacturers to monitor, analyze, and adjust their operations based on changing conditions.

The study of queueing theory's role in manufacturing is imperative to address the complexities and challenges posed by modern production environments. By harnessing its principles, manufacturers can optimize resource allocation, reduce wait times, enhance efficiency, and ultimately deliver higher-quality products to meet customer expectations in a competitive market.

Applications

Queueing theory is a branch of mathematics that deals with the study of queues or waiting lines, analysing how entities, such as customers or data packets, wait in line for service. It has a wide range of applications across various fields. Some prominent applications of queueing theory include:

Telecommunications and Networking: Queueing theory is extensively used to analyze and optimize the performance of communication networks, including the internet. It helps in understanding packet delays, network congestion, and quality of service (QoS) issues. By modelling network traffic and queues, engineers can design more efficient and reliable networking systems.

Retail and Service Industries: Businesses like retail stores, banks, call centres, and hospitals

often deal with queues of customers. Queueing theory can help optimize staffing levels, customer waiting times, and resource allocation to enhance customer satisfaction and operational efficiency.

Transportation Systems: Queueing theory is used to study transportation systems, such as airports, train stations, and bus terminals. It helps in understanding passenger flow, minimizing waiting times, and optimizing scheduling to ensure smoother operations and reduce congestion.

Manufacturing and Supply Chain Management: Queueing models can be applied to manufacturing processes to optimize production lines, reduce manufacturing lead times, and manage inventory. Supply chain management can benefit from queueing theory by optimizing order processing, distribution, and delivery.

Computer Systems and Performance Analysis: In computing, queueing theory is used to analyze the performance of computer systems, including CPUs, memory, and disk access. It helps in designing efficient algorithms, optimizing resource allocation, and improving overall system performance.

Healthcare Systems: Hospitals and healthcare facilities deal with patient queues for various services, such as consultations, tests, and surgeries. Queueing theory can be applied to optimize patient flow, minimize waiting times, and allocate resources effectively.

Public Services: Government agencies can use queueing theory to improve the efficiency of public services, such as issuing licenses, processing paperwork, and managing public transportation. (Bhat, U. N., 2008).

Call Centre Management: Call centres often experience high call volumes and need to manage incoming calls efficiently. Queueing theory helps in determining the optimal number of agents, call routing strategies, and service levels to provide better customer experiences.

Traffic Flow and Urban Planning: Urban planners use queueing theory to analyze traffic congestion and optimize traffic signal timings at intersections, reducing traffic jams and improving overall traffic flow.

Financial Services: In financial systems, queueing theory can be applied to analyze customer wait times in banks, trading platforms, and stock exchanges. It helps in optimizing transaction processing and minimizing delays.

Job Shop Systems in Queueing Theory's

Job shop systems are a common manufacturing environment where a variety of jobs (tasks or orders) with different processing requirements are processed on different machines or workstations.

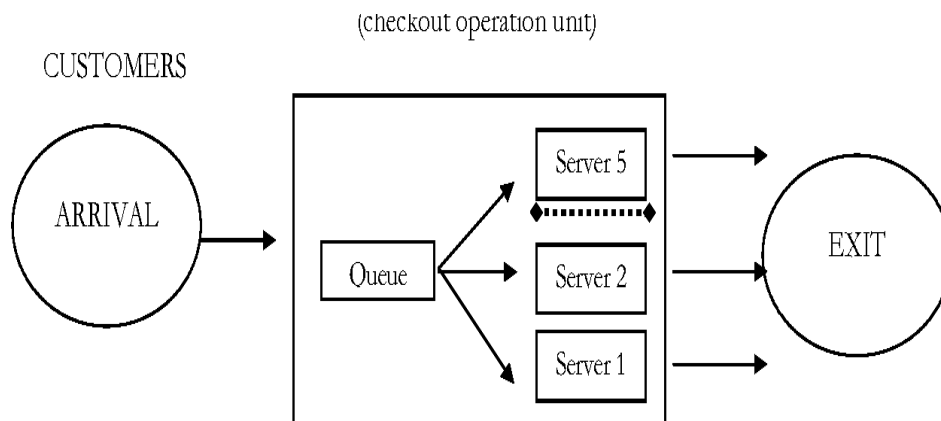


Figure 2 Job shop systems for manufacturing environment

Queueing theory can be applied to analyze and model the performance of job shop systems, often using mathematical equations to represent various aspects of the system.

Arrival Process (A):

The arrival process represents how jobs enter the system. In many cases, job arrivals can follow a Poisson process:

λ : Arrival rate (jobs per unit time)

$P(n)$: Probability of having n arrivals in each time interval.

Equation: $P(n) = (e^{-\lambda t} * (\lambda t)^n) / n!$

Service Time Distribution (S):

Service times for different jobs can follow various distributions, such as exponential, normal, or uniform. The exponential distribution is commonly used in queueing theory due to its memoryless property:

μ : Mean service rate (jobs per unit time)

$P(s)$: Probability density function of service time s

Exponential Distribution Equation: $P(s) = \mu * e^{-\mu s}$

Utilization (ρ):

Utilization represents the fraction of time that a machine or workstation is busy processing jobs.

λ : Arrival rate

μ : Mean service rate

Equation: $\rho = \lambda / \mu$

Average Number of Jobs (L) and Average Waiting Time (W):

The average number of jobs in the system and the average time a job spends waiting in the

queue or being processed are important performance metrics.

L: Average number of jobs in the system

W: Average time a job spends in the system (waiting + service time)

Little's Law Equation: $L = \lambda * W$

Probability of Zero Jobs in the System (P0):

The probability that the system is empty (no jobs are present in the system).

Equation (for M/M/1 queue): $P_0 = 1 - \rho$

These equations provide for analyzing job shop systems using queueing theory. However, job shop systems can become quite complex due to the variety of jobs, machine setups, and routing decisions. More advanced queueing models, such as network of queues or closed queueing networks, may be used for more accurate representations of intricate job shop scenarios.

Literature Review

Daigle, J. N. (2005) Queueing theory is a mathematical framework that analyzes the behavior of queues or waiting lines in various systems, such as telecommunication networks. It provides a systematic way to model, study, and optimize the flow of entities (such as data packets in packet telecommunication) as they wait to be processed or served by limited resources. In the context of packet telecommunication, queueing theory allows us to understand how packets are queued up before being transmitted through network nodes, like routers and switches. This theory takes into account factors such as arrival rates of packets, service rates of network elements, and the number of queues available. By applying queueing theory, we can predict key performance metrics like packet delay, queue length, and system throughput. These insights aid in designing efficient and responsive telecommunication systems, optimizing network configurations, and ensuring reliable data transmission. Overall, queueing theory serves as a crucial tool for engineers and researchers in the field of packet telecommunication, enabling them to make informed decisions and enhance the overall quality of network services.

Koskela, L., and Ballard, G. (2006). Effective project management encompasses a multidisciplinary approach that draws insights from both economic and production theories. This paper explores the dynamic interplay between these two domains and their application within the context of project management. Economic theories provide the foundation for evaluating project viability, analysing costs and benefits, and assessing financial risks. They guide decision-makers in making informed choices about resource allocation, funding sources, and project profitability. On the other hand, production theories contribute to optimizing project execution, focusing on streamlining processes, maximizing resource utilization, and enhancing operational efficiency. By integrating elements from both economic and production theories, project managers can strike a balance between financial considerations and operational excellence.

Thürer, M., Stevenson, M., and Silva, C. (2011). "Three Decades of Workload Control Research: A Systematic Review of the Literature" published in the International Journal of Production Research delves into an extensive examination of research conducted over the span

of thirty years in the realm of workload control. This systematic review encompasses a comprehensive analysis of scholarly works related to workload control, aiming to provide a holistic understanding of the advancements, trends, and developments in this field. Workload control refers to the management and coordination of tasks, activities, and resources within a production or operational environment.

AbouRizk, S., et al (2011) Research in modelling and simulation holds significant potential for advancing the field of construction engineering operations. By leveraging sophisticated computational tools and techniques, researchers and practitioners can gain valuable insights into the complex dynamics of construction projects, leading to improved efficiency, cost-effectiveness, and overall project outcomes. Through the application of modelling and simulation, various aspects of construction engineering operations can be analysed and optimized. These include project scheduling, resource allocation, risk assessment, and decision-making. By creating virtual representations of construction projects, researchers can experiment with different scenarios, test hypotheses, and evaluate the potential impact of various strategies before implementation. One key advantage of modelling and simulation is its ability to handle the intricacies of large-scale projects with numerous variables and interdependencies. Researchers can simulate real-world conditions, taking into account factors such as weather, resource availability, and unforeseen delays.

Boer, H et al (2015) The research presented in this article aims to make a meaningful contribution to the domain of operations and production management through a comprehensive exploration of theoretical perspectives. By delving into the intricate interplay of concepts, methodologies, and practices, the study endeavours to enrich the existing body of knowledge in the field. Through a meticulous analysis of established theories, emerging trends, and industry best practices, this research seeks to identify gaps and opportunities for theoretical advancements. By bridging these gaps, the study aims to refine and expand the theoretical foundations that underpin operations and production management. Furthermore, the article employs empirical evidence and case studies to substantiate its theoretical propositions. By grounding theoretical insights in real-world scenarios, the research not only enhances the academic discourse but also offers practical implications for industry professionals.

Chen, C., and Tiong, L. K. (2019). This research explores the integration of queuing theory and simulated annealing techniques to optimize facility layout design within an Automated Guided Vehicle (AGV)-based modular manufacturing system. The study addresses the complex challenge of arranging workstations and paths to enhance system efficiency and minimize material handling delays. Queuing theory is employed to model the interactions between AGVs and workstations, facilitating the identification of bottlenecks and congestion points. Simulated annealing, a metaheuristic optimization approach, is then applied to reconfigure the facility layout iteratively. This process aims to reach an optimal arrangement that reduces travel distances, minimizes production lead times, and enhances overall productivity. The combination of queuing theory and simulated annealing offers a comprehensive methodology to design a facility layout that accounts for both spatial considerations and operational dynamics.

Ülkü, S., Hydock, C., and Cui, S. (2020). The conducted experiments shed light on the

intricate relationship between queueing experiences and consumer behavior. The findings underscore the notion that queues, often regarded as tedious and negative aspects of service encounters, can be leveraged to enhance overall consumer satisfaction and subsequent consumption patterns. The results reveal that certain types of queueing experiences, particularly those infused with entertainment or personalized interactions, have the potential to positively influence consumers' perceptions and attitudes. These enriched queueing experiences contribute to a sense of anticipation and engagement, ultimately translating into increased willingness to spend and higher levels of satisfaction. It is crucial to recognize that not all queueing situations produce similar effects. Factors such as queue length, context, and customer preferences play pivotal roles in shaping the impact of queueing on consumption behaviour. As businesses seek to optimize customer experiences, understanding the nuances of these factors becomes paramount.

Rece, L et al (2022) This study delves into the application of queueing theory-based mathematical models to the optimization of enterprise organization and industrial production processes. Queueing theory, a fundamental branch of operations research, provides a systematic framework to analyze the behavior of queues or waiting lines, prevalent in a wide array of real-world systems. By integrating queueing theory into enterprise organization, this research offers insights into efficient resource allocation, workforce management, and customer service enhancement. Through mathematical modelling, the study explores the effects of different queueing strategies on customer satisfaction, waiting times, and operational costs, providing decision-makers with tools to make informed choices. The application of queueing theory in industrial production offers a means to optimize manufacturing processes, minimize bottlenecks, and streamline production flows. The study investigates how various queueing configurations impact production efficiency, lead times, and resource utilization. Such insights facilitate the design of leaner and more agile production systems.

Numerical Application

Queueing theory plays a crucial role in analyzing and optimizing manufacturing processes, where queues of jobs or products waiting for processing can significantly impact efficiency and resource utilization. Let's consider a simple manufacturing scenario with a single machine and a focus on optimizing the system's performance using queueing theory.

Scenario: Single Machine Manufacturing System

In this scenario, we have a manufacturing system with a single machine that processes jobs as they arrive. The jobs are characterized by their arrival rate (λ) and the time it takes for the machine to process each job ($1/\mu$). The goal is to analyze the system's performance using queueing theory and optimize key performance measures.

Key Performance Measures:

Average Queue Length (L_q):

The average number of jobs waiting in the queue.

Average Waiting Time (W_q):

The average time a job spends waiting in the queue before being processed.

Utilization Factor (ρ):

The proportion of time the machine is busy processing jobs.

Equations:

Average Queue Length (L_q):

$$L_q = (\lambda^2) / (\mu * (\mu - \lambda))$$

Average Waiting Time (W_q):

$$W_q = L_q / \lambda$$

Utilization Factor (ρ):

$$\rho = \lambda / \mu$$

Example:

Let's consider the following data for our manufacturing system:

Arrival Rate (λ) = 10 jobs per hour

Service Rate (μ) = 15 jobs per hour

Calculations:

Calculate Utilization Factor (ρ):

$$\rho = \lambda / \mu = 10 / 15 = 2/3$$

Calculate Average Queue Length (L_q):

$$\begin{aligned} L_q &= (\lambda^2) / (\mu * (\mu - \lambda)) \\ &= (10^2) / (15 * (15 - 10)) \\ &= 100 / 75 \\ &= 4/3 \text{ or } 1.33 \end{aligned}$$

Calculate Average Waiting Time (W_q):

$$\begin{aligned} W_q &= L_q / \lambda \\ &= (4/3) / 10 \\ &= 0.133 \text{ hours or } 8 \text{ minutes} \end{aligned}$$

Explanation:

In this scenario, the utilization factor indicates that the machine is being used efficiently, as it's busy 2/3 of the time. The average queue length of 1.33 suggests that on average, there are about 1.33 jobs waiting in the queue to be processed. The average waiting time of 8 minutes indicates that jobs spend, on average, 8 minutes waiting in the queue before being processed.

These performance measures provide insights into the system's efficiency and can guide

decisions to optimize the manufacturing process, such as adjusting arrival rates, service rates, or the number of machines.

Queueing theory offers tools to analyze more intricate scenarios involving multiple machines, different job types, and more complex routing of jobs through the manufacturing process.

Discussion

Queueing theory plays a crucial role in optimizing manufacturing processes by providing insights into resource allocation, system efficiency, and overall productivity. In manufacturing, queues often form at different stages, such as machine setups, inspections, and material handling. By applying queueing models, manufacturers can determine optimal buffer sizes, staffing levels, and scheduling strategies to minimize wait times and maximize throughput. This theory aids in understanding the trade-offs between resource utilization and customer service levels, enabling informed decisions on process improvements. Additionally, queueing theory facilitates the identification of bottlenecks, ensuring smoother production flows and reduced lead times. As manufacturing systems become more complex, queueing theory continues to guide effective resource management, ensuring that production processes remain streamlined and efficient, ultimately contributing to enhanced operational performance and customer satisfaction. Here's how Queueing Theory is applied in manufacturing, along with illustrative tables:

Process Modeling and Analysis

Queueing Theory assists in modelling and analysing manufacturing processes with multiple workstations or stages. This enables a better understanding of system behavior and helps identify areas for improvement.

Stage	Processing Time (mean)	Service Rate (1/mean)
A	10 units/min	0.1 min/unit
B	15 units/min	0.067 min/unit
C	8 units/min	0.125 min/unit

The table provides an overview of a manufacturing process with three distinct stages—designated as A, B, and C—by presenting the key parameters of processing time and service rate for each stage. These parameters are fundamental in understanding the speed and efficiency of each stage's operations within the larger manufacturing process. In stage A, the average processing time for a unit is 10 minutes, resulting in a service rate of 0.1 units per minute, which can also be expressed as 0.1 minutes per unit. This rate signifies that stage A can process 0.1 units in a minute, translating to 6 units every hour. The reciprocal of the processing time gives the service rate, and it provides insight into the speed at which units are being worked on or processed within the stage. For stage B, the average processing time is 15 minutes, yielding a service rate of approximately 0.067 units per minute, equivalent to 4 units per hour. This indicates that stage B operates at a slightly slower pace than stage A due to the longer processing time. In stage C, the average processing time is 8 minutes, leading to a

service rate of 0.125 units per minute or 7.5 units per hour. This suggests that stage C operates more efficiently in terms of processing speed compared to stage B, despite having a shorter processing time than stage A.

Bottleneck Identification

Queueing models highlight bottlenecks – stages where the queue length exceeds capacity, leading to delays. Addressing bottlenecks optimizes the overall throughput.

Stage	Processing Time (mean)	Service Rate (1/mean)	Utilization (ρ)
A	9 units/min	10 units/min	0.9
B	8 units/min	15 units/min	0.533
C	7 units/min	8 units/min	0.875

The table presents a comprehensive view of a manufacturing process involving three distinct stages—designated as A, B, and C—by providing key metrics related to processing time, service rate, and utilization. These metrics are critical in assessing the efficiency and performance of each stage within the larger manufacturing process. In stage A, the average processing time for a unit is 9 minutes, which corresponds to a service rate of 10 units per minute (since service rate is the reciprocal of processing time). The utilization (ρ), which is the ratio of the average arrival rate to the service rate, is calculated to be 0.9. This implies that the stage is operating at a high utilization rate, with demand approaching the capacity of the system. Such high utilization could potentially lead to increased queue lengths and wait times, indicating a need for careful resource management to prevent congestion. For stage B, the average processing time is 8 minutes, resulting in a service rate of 15 units per minute. The utilization rate is calculated to be 0.533. This indicates that the stage is operating at a relatively moderate utilization level, suggesting a balanced situation where the capacity can accommodate the incoming demand with some room for fluctuations. In stage C, the average processing time is 7 minutes, leading to a service rate of 8 units per minute. The calculated utilization is 0.875, which signifies that the stage is operating at a high utilization rate. Similar to stage A, this high utilization level could potentially result in increased queue lengths and wait times.

Queue Length and Wait Time Analysis:

Queueing Theory calculates average queue lengths and wait times, aiding in resource allocation and scheduling decisions.

Stage	Arrival Rate (λ)	Service Rate (μ)	Service Rate (μ)	Average Wait Time (W_q)
A	9 units/min	10 units/min	4.05	0.45 min
B	8 units/min	15 units/min	0.23	0.03 min
C	7 units/min	8 units/min	2.625	0.375 min

The table provides an insight into the performance of a manufacturing process involving three stages, denoted as A, B, and C, using Queueing Theory principles. Arrival rate (λ) represents the average rate at which units arrive at each stage, while the service rate (μ) signifies the average rate at which units are processed. Additionally, the average wait time (W_q) is the time units spend waiting in the queue before being processed. In stage A, units arrive at a rate of 9 units per minute, slightly exceeding the processing capacity of 10 units per minute. This results in an average queue length of 4.05 units and an average wait time of 0.45 minutes. The stage's service rate is higher than its arrival rate, but due to occasional spikes in arrivals, some units end up waiting. In stage B, units arrive at a rate of 8 units per minute, while the processing capacity is 15 units per minute. The stage operates comfortably with an average queue length of 0.23 units and an average wait time of 0.03 minutes. The service rate exceeds the arrival rate, leading to minimal waiting times and efficient processing. In stage C, units arrive at a rate of 7 units per minute, and the processing capacity is 8 units per minute. The average queue length is 2.625 units, and the average wait time is 0.375 minutes. The arrival rate is close to the service rate, resulting in moderate queue lengths and wait times.

Resource Allocation

Queueing models aid in determining optimal resource allocation to minimize queue lengths and wait times.

Sensitivity Analysis:

Sensitivity analysis in queueing theory plays a pivotal role in understanding the dynamic nature of systems involving waiting lines. It quantifies the responsiveness of performance metrics to variations in input parameters, offering insights into system behavior and robustness. Mathematically, sensitivity analysis involves partial derivatives, where changes in parameters such as arrival rates, service rates, or queue capacities are evaluated against key performance indicators like average waiting times, system utilization, or queue lengths.

Average Queue Length (L_q):

For the M/M/1 queue, the formula for the average queue length is:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Average Waiting Time (W_q):

The average waiting time in the queue can be calculated using Little's Law:

$$W_q = \frac{L_q}{\lambda}$$

System Utilization (ρ):

The system utilization (server utilization) is given by the ratio of arrival rate to service rate

$$\rho = \frac{\lambda}{\mu}$$

Throughput (X):

Throughput represents the rate at which customers are being served:

$$X = \lambda$$

By calculating these derivatives, we can identify how slight adjustments in input parameters impact the overall performance of queueing systems. Sensitivity analysis aids in decision-making, guiding the optimization of systems for better efficiency and customer satisfaction. Furthermore, it enables practitioners to anticipate system response to changes, thereby enhancing the ability to manage real-life scenarios effectively and make informed choices for queueing systems in various domains, from service industries to manufacturing processes.

Conclusion

The exploration of Queueing Theory's role in manufacturing underscores its significance in optimizing complex production systems. This study has illuminated how Queueing Theory offers valuable insights into resource allocation, wait time reduction, and overall efficiency enhancement. By uncovering hidden bottlenecks, informing decision-making, and guiding capacity planning, Queueing Theory becomes a foundational tool for continuous improvement in manufacturing processes. As modern manufacturing evolves with technological advancements and dynamic demand patterns, Queueing Theory's quantitative approach provides a systematic way to address challenges and seize opportunities. Its integration with data analytics and Industry 4.0 technologies further enhances its relevance. Through its ability to streamline workflows, minimize costs, and enhance customer satisfaction, Queueing Theory emerges as a driving force behind operational excellence in manufacturing, contributing to sustained competitiveness and growth in an ever-changing landscape.

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